التكامل غير المحدد

أولاً: جدول التكاملات الأساسية :

$$\boxed{01}\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\boxed{02}\int x^{\frac{m}{n}} dx = \frac{x^{\frac{m}{n}+1}}{\frac{m}{n}+1} + C$$

$$\boxed{03}\int e^x dx = e^x + C$$

$$\boxed{04}\int a^x dx = \frac{a^x}{lna} + C$$

$$\boxed{05}\int \frac{dx}{x} = ln|x| + C$$

$$\boxed{06}\int cosxdx = sinx + C$$

$$\boxed{07}\int sinxdx = -cosx + C$$

$$\boxed{08}\int \frac{dx}{cos^2 x} = \int(1 + tan^2 x) = tanx + C$$

$$\boxed{09}\int \frac{dx}{sin^2 x} = \int(1 + cot^2 x) = -cotx + C$$

$$\boxed{10}\int sinhxdx = coshx + C$$

$$\boxed{11}\int coshxdx = sinhx + C$$

$$\boxed{12}\int \frac{dx}{cosh^2x} = \int(1 - tanh^2x) = tanhx + C$$

$$\boxed{13}\int \frac{-dx}{sinh^2x} = \int(1 - coth^2x) = cothx + C$$

$$\boxed{14}\int \frac{dx}{\sqrt{1-x^2}} = \begin{cases} +arcsinx + C \\ -arccosx + C \end{cases}$$

$$\boxed{15}\int \frac{dx}{1+x^2} = \begin{cases} +arctanx + C \\ -arccotx + C \end{cases}$$

$$\boxed{16}\int \frac{dx}{\sqrt{x^2+1}} = sinh^{-1}x + C$$

$$\boxed{17}\int \frac{dx}{\sqrt{x^2-1}} = cosh^{-1}x + C$$

$$\boxed{18}\int \frac{dx}{1-x^2} = \begin{cases} +tanh^{-1}x + C \\ +coth^{-1}x + C \end{cases}$$

تمارين : اوجد التكاملات الآتية :

❶ $I = \int \frac{x^5+\sqrt{x}+1}{x^3}dx$

$$I = \int \left(\frac{x^5}{x^3} + \frac{\sqrt{x}}{x^3} + \frac{1}{x^3}\right)dx = \int \left(x^2 + x^{-\frac{5}{2}} + x^{-3}\right)dx$$

$$I = \frac{x^3}{3} + \frac{x^{-\frac{3}{2}}}{-\frac{3}{2}} + \frac{x^{-2}}{-2} + C$$

$$\boxed{I = \frac{x^3}{3} - \frac{2}{3\sqrt{x^3}} - \frac{1}{2x^2} + C}$$

❷ $I = \int \frac{e^{tanx}}{cos^2x} dx$

$Let: u = tanx \rightarrow du = \frac{dx}{cos^2x}$

$I = \int e^u du = e^u + C \rightarrow \boxed{I = e^{tanx} + C}$

❸ $I = \int \frac{3^{(1+\sqrt{x})}}{\sqrt{x}} dx$

$Let: u = 1 + \sqrt{x} \rightarrow du = \frac{dx}{2\sqrt{x}} \rightarrow \frac{dx}{\sqrt{x}} = 2du$

$I = 2\int 3^u du = \frac{3^u}{ln3} + C \rightarrow \boxed{I = \frac{2}{ln3}\left[3^{(1+\sqrt{x})}\right] + C}$

❹ $I = \int \frac{1+tan^2x}{10+tanx} dx$

$Let: u = 10 + tanx \rightarrow du = (1 + tan^2x)dx$

$I = \int \frac{du}{u} = ln|u| + C \rightarrow \boxed{I = ln|10 + tanx| + C}$

❺ $I = \int \frac{1+sin^3x}{sin^2x} dx$

$I = \int \left(\frac{1}{sin^2x} + \frac{sin^3x}{sin^2x}\right) dx = \int \left(sinx + \frac{1}{sin^2x}\right) dx$

$\boxed{I = cosx - cotx + C}$

❻ $I = \int tan^3x \cdot dx$

$I = \int tanx \cdot tan^2x dx = \int tanx[(1 + tan^2x) - 1]dx$

$I = \int tanx \cdot (1 + tan^2x)dx - \int tanx \cdot dx$

$I = \int tanx \cdot (1 + tan^2x)dx - \int \frac{sinx}{cosx} dx$

$$\boxed{I = \frac{1}{2}tan^2x + ln|cosx| + C}$$

❼ $I = \int \frac{e^x}{\sqrt{1-e^{2x}}} dx = \int \frac{e^x}{\sqrt{1-(e^x)^2}} dx$

$Let: u = e^x \rightarrow du = e^x \cdot dx$

$I = \int \frac{du}{\sqrt{1-(u)^2}} = arcsin(u) + C$

$$\boxed{I = arcsin(e^x) + C}$$

❽ $I = \int \frac{sin2x}{\sqrt{1+sin^4x}} dx$

$Let: u = sin^2x \rightarrow du = 2sinx \cdot cosx \cdot dx = sin2x \cdot dx$

$I = \int \frac{du}{\sqrt{1+(u)^2}} = sinh^{-1}(u) + C$

$$\boxed{I = sinh^{-1}(sinx) + C}$$

❾ $I = \int e^{3x} \cdot sinh(2x) \cdot dx$

$$I = \int e^{3x} \left(\frac{e^{2x}-e^{-2x}}{2}\right) dx = \int \frac{e^{5x}-e^{x}}{2} dx$$

$$I = \frac{1}{2}\int (e^{5x} - e^{x})dx = \frac{1}{2}\left(\frac{1}{5}e^{5x} - e^{x}\right) + C$$

$$\boxed{I = \frac{1}{10}e^{5x} - e^{x} + C}$$

❿ $I = \int \frac{(1+x)^2}{x(1+x^2)} dx$

$$I = \int \frac{(1+x)^2}{x(1+x^2)} dx = \int \frac{1+2x+x^2}{x(1+x^2)} dx = \int \frac{(1+x^2)+2x}{x(1+x^2)} dx$$

$$I = \int \frac{(1+x^2)}{x(1+x^2)} dx + \int \frac{2x}{x(1+x^2)} dx$$

$$I = \int \frac{dx}{x} + 2\int \frac{dx}{(1+x^2)}$$

$$\boxed{I = ln|x| + 2arctanx + C}$$