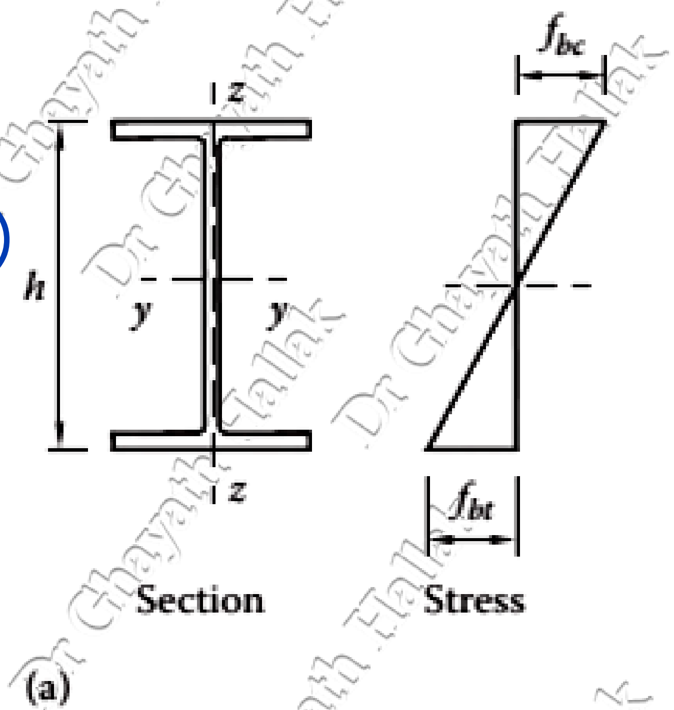
BENDING STRESSES AND MOMENT CAPACITY

1 Elastic theory

1. Uniaxial bending

From the Mechanics of Material course (Symmetrical bending)



Max stress $$f_{bt} = f_{bc} = \frac{M_y}{I_y} Z = \frac{M_y}{W_{el,y}}$$

Moment capacity $$M_c = \sigma_b W_{el,y}$$

$W_{el,y}$ *elastic section modulus about the majour axis*

The elastic design resistance for bending about one principal axis is determined in Clause 5.2.5.2 of EN 1993-1-1 as follows:

$$M_{el,Rd} = \frac{W_{el} f_y}{\gamma_{M0}}$$

BENDING STRESSES AND MOMENT CAPACITY

1 Elastic theory

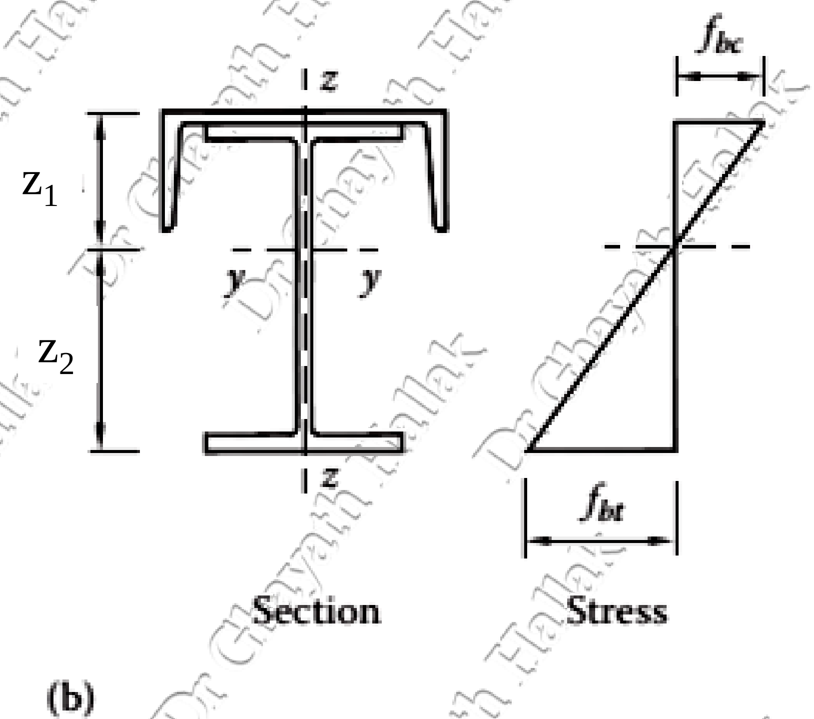
1. Uniaxial bending

For the asymmetrical crane beam section:

$W_{el,y1} = I_y / z_1$ = modulus of section for top flange.

$W_{el,y2} = I_y / z_2$ = modulus of section for bottom flange.((MIN))

z_1, z_2 = distance from centroid to top and bottom fibres.



Stress at top $$f_{bc} = \frac{M_y}{W_{el,y1}}$$

Stress at bottom $$f_{bt} = \frac{M_y}{W_{el,y2}}$$

max → f_{bt}; min → $W_{el,y2}$

Moment capacity: $$M_{el,Rd} = \frac{W_{el,y2}\, f_y}{\gamma_{M0}}$$

min → $W_{el,y2}$

BENDING STRESSES AND MOMENT CAPACITY

1 Elastic theory

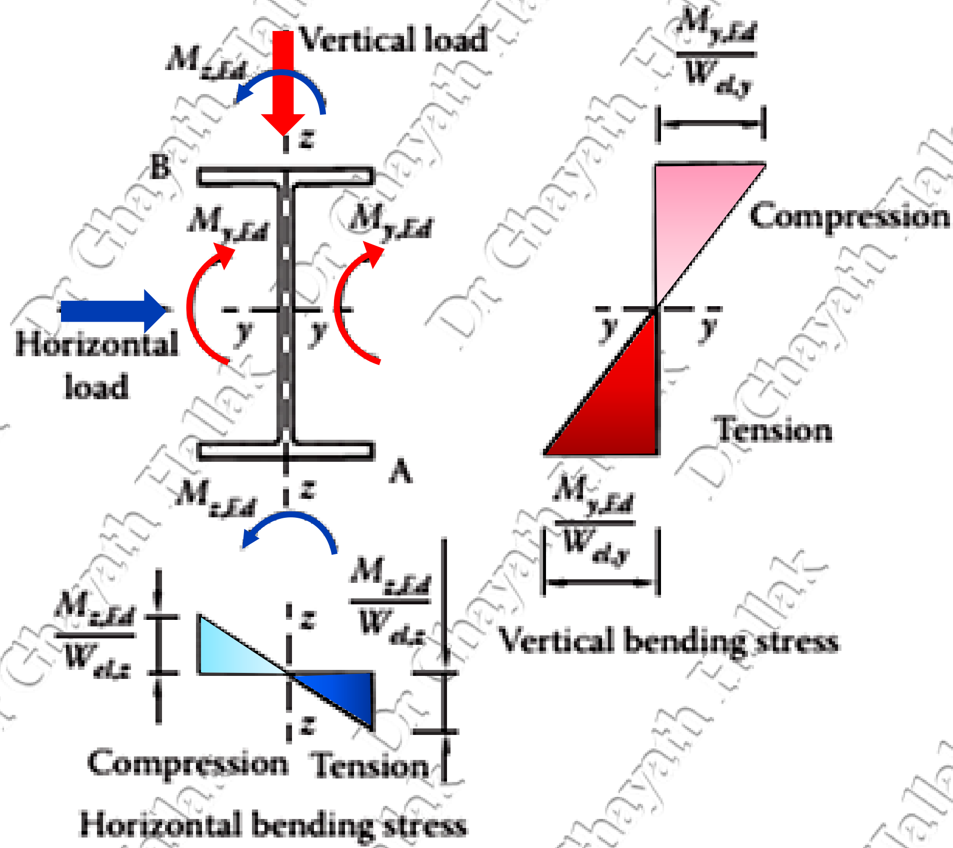
2. Biaxial bending

My,Ed = design bending moment about the *y–y* axis.

Mz,Ed = design bending moment about the *z–z* axis.

Wel,y = elastic section modulus for the *y–y* axis.

Wel,z = elastic section modulus for the *z–z* axis.



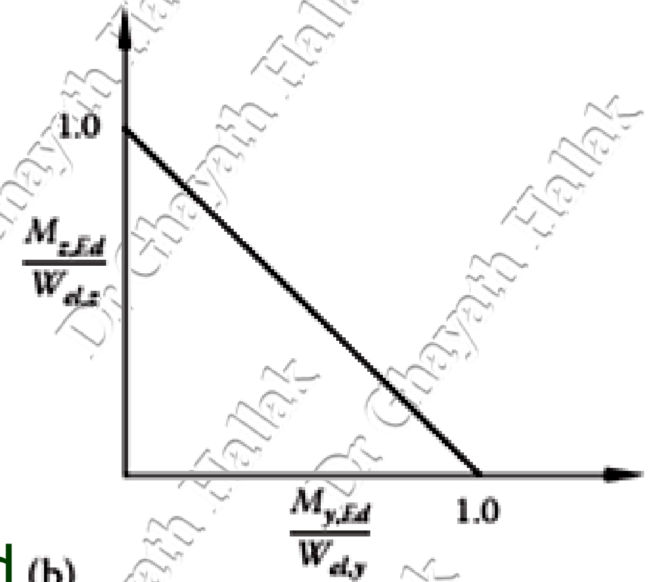
The maximum Stress at A or B

$$f_A = f_B = \frac{M_{y,Ed}}{W_{el,y}} + \frac{M_{z,Ed}}{W_{el,z}} \le \frac{f_y}{\gamma_{M0}}$$

BENDING STRESSES AND MOMENT CAPACITY

1 Elastic theory

2. Biaxial bending



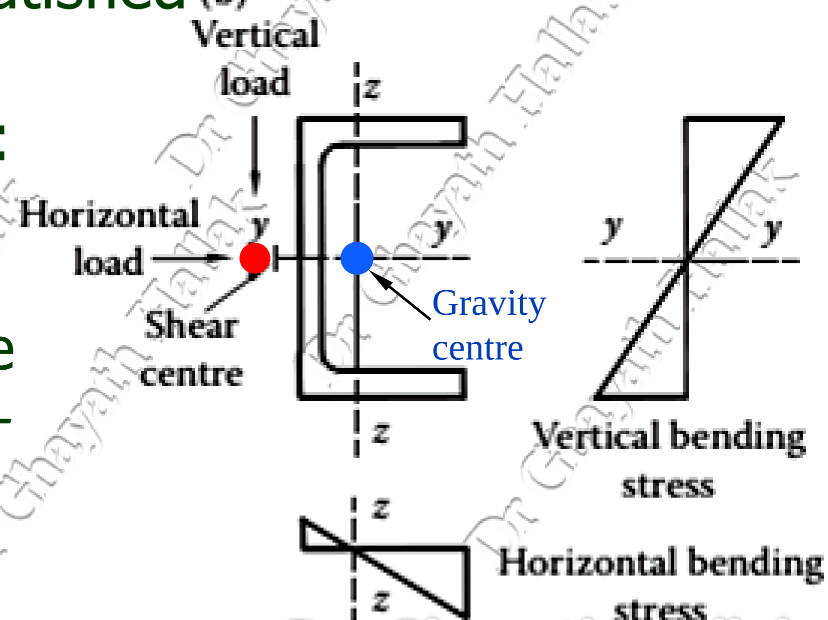
The elastic design resistance for bending

$$M_{el,y,Rd} = \frac{W_{el,y}\, f_y}{\gamma_{M0}} \quad , \quad M_{el,z,Rd} = \frac{W_{el,z}\, f_y}{\gamma_{M0}}$$

$$\frac{M_{y,Ed}}{M_{el,y,Rd}} + \frac{M_{z,Ed}}{M_{el,z,Rd}} \leq 1$$

Should be satisfied

(b)



For asymmetrical beam sections:

In channel sections, the vertical load must be applied through the shear centre for bending in the free member to take place about the *y*–*y* axis; otherwise, twisting and biaxial bending occurs.

BENDING STRESSES AND MOMENT CAPACITY

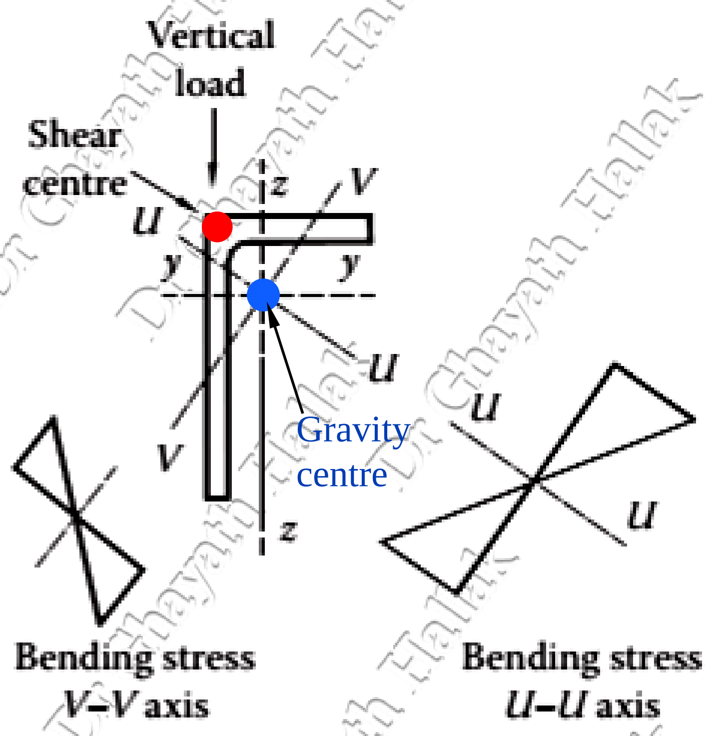
1 Elastic theory

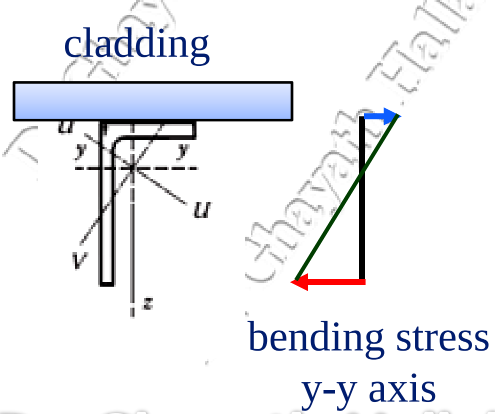
2. Biaxial bending

For asymmetrical beam sections:

In unequal angles, bending takes place about the principle axes *U-U* and *V-V* in the free member when the load is applied through the shear centre.

When the angle is used as a purlin, the cladding restrains the member so that it bends about the *y–y* axis.



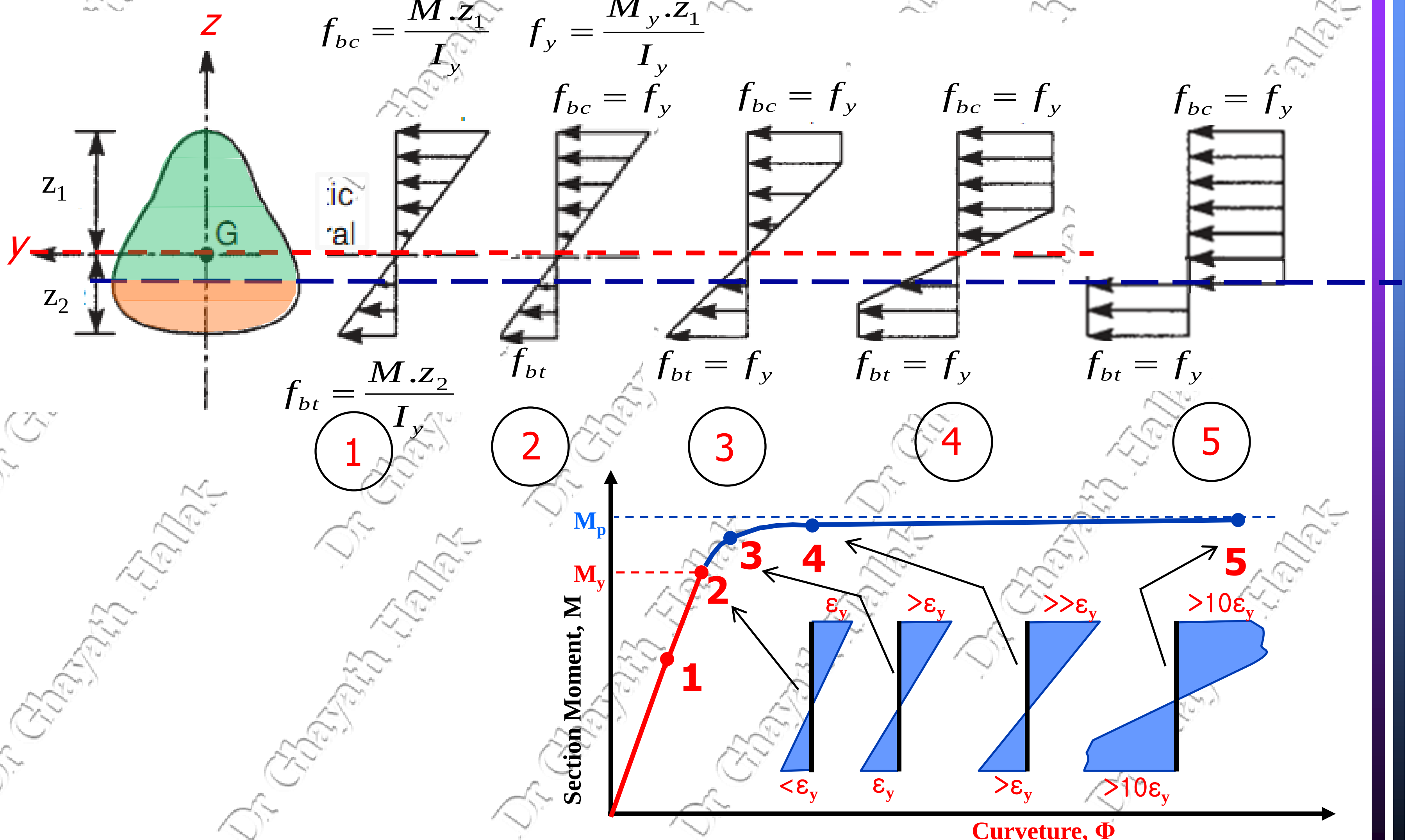


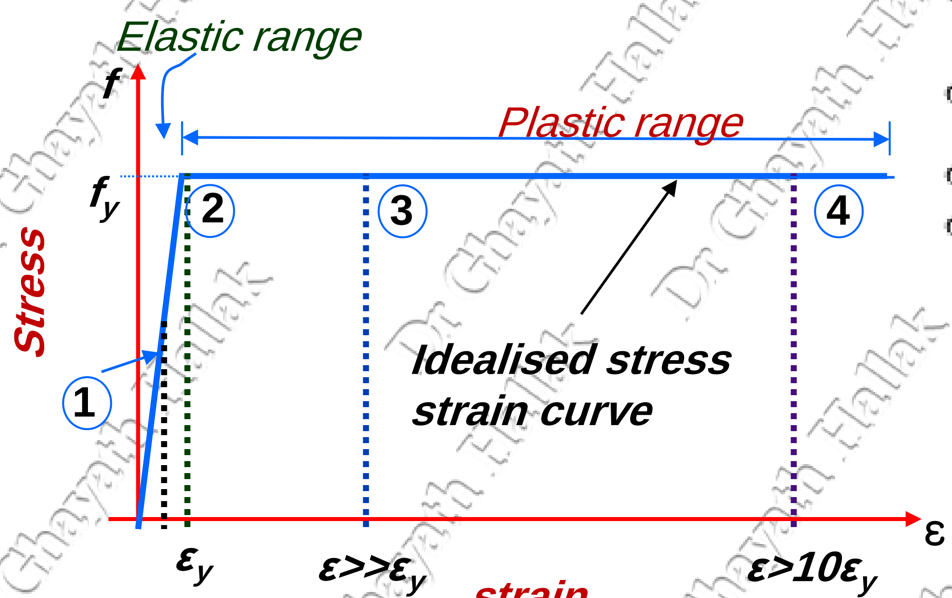
BENDING STRESSES AND MOMENT CAPACITY

2 Plastic theory 1. uniaxial bending

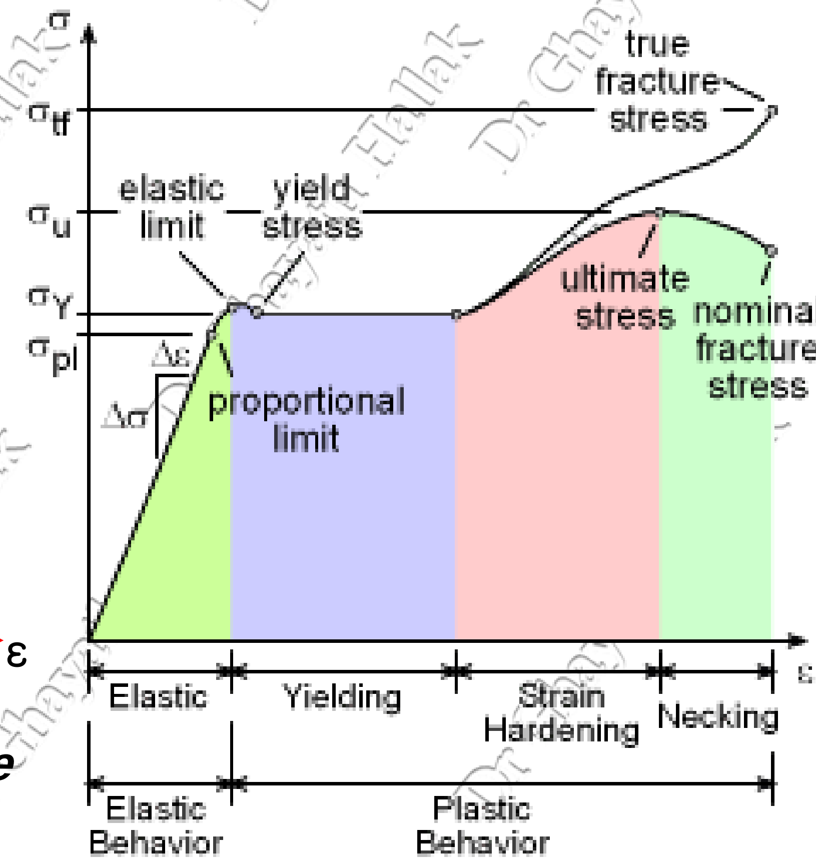
$$f_{bc} = \frac{M.z_1}{I_y} \qquad f_y = \frac{M_y.z_1}{I_y}$$

$$f_{bt} = \frac{M.z_2}{I_y}$$

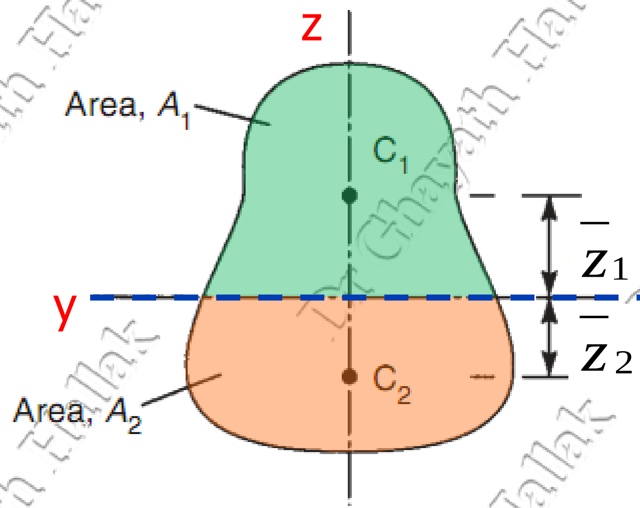


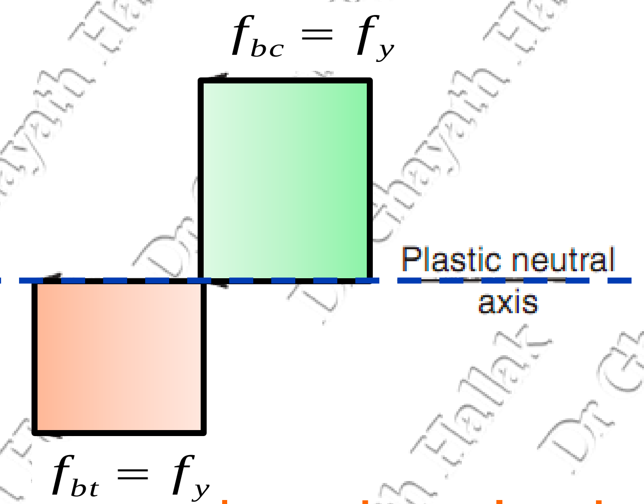


Idealized elasto- plastic stress stain curve for the purpose of design



2 Plastic theory 1. uniaxial bending





A_2 the area of the beam section below the plastic neutral axis (tensile)

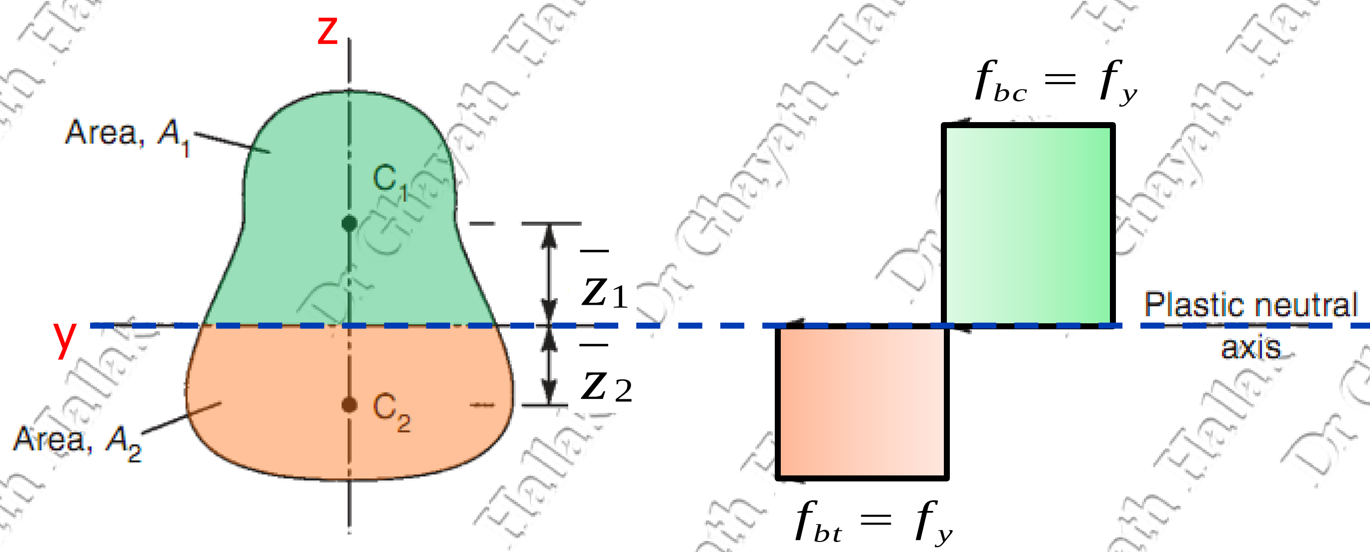
A_1 the area of the beam section above the plastic neutral axis (compressive)

Since M_P is a pure bending moment ➡ the total direct load on the beam section must be zero.

$$f_{bt} \times A_2 = f_{bc} \times A_1 \Rightarrow f_y \times A_2 = f_y \times A_1 \Rightarrow A_2 = A_1$$

$$if \;\; A = A_1 + A_2 \Rightarrow A_2 = A_1 = \frac{A}{2}$$

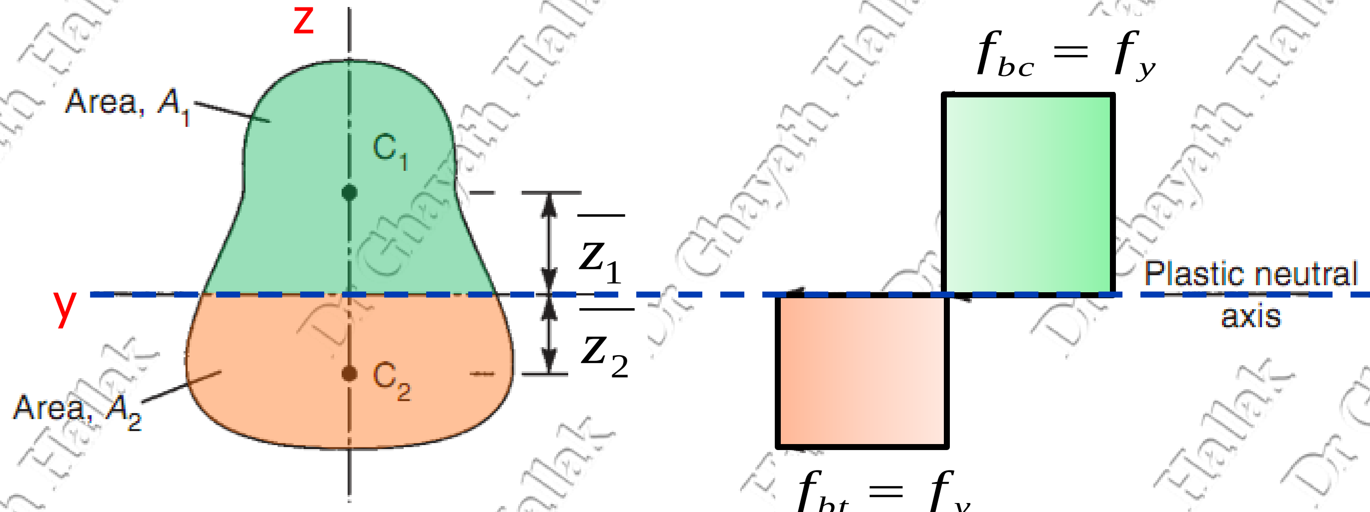
2 Plastic theory 1. uniaxial bending



we see that the plastic neutral axis divides the beam section into two equal areas.

Clearly for doubly symmetrical sections or for singly symmetrical sections in which the plane of the bending moment is perpendicular to the axis of symmetry, the elastic and plastic neutral axes coincide.

2 Plastic theory 1. uniaxial bending



The plastic moment, M_P, can now be found by taking moments of the resultants of the tensile and compressive stresses about the Plastic neutral axis. These stress resultants act at the centroids C_1 and C_2 of the areas A_1 and A_2, respectively.

$$M_p = f_y \times A_2 \times \bar{z}_2 + f_y \times A_1 \times \bar{z}_1 \Rightarrow M_p = f_y \times \frac{A}{2} \times \left(\bar{z}_1 + \bar{z}_2\right)$$

$$M_p = W_{pl} \times f_y \quad where \quad W_{pl} = \frac{A}{2} \times \left(\bar{z}_1 + \bar{z}_2\right)$$

2 Plastic theory -1. uniaxial bending

$$W_{pl} = \Sigma \ first\ moment\ of\ \ area\ about\ the\ plastic\ NA)$$

W_{Pl} is known as the plastic modulus of the cross section. Note that the elastic modulus, W_{el}, has two values for a beam of singly symmetrical cross section (bending takes place about the centroidal axis) whereas the plastic modulus is single-valued. (bending takes place about the equal area axis-plastic neutral axis)

SHAPE FACTOR

The ratio of the plastic moment of a beam to its yield moment is known as the *shape factor*. Thus

$$Shape\ \ factor = \frac{M_p}{M_y} = \frac{W_{pl} \times f_y}{W_{el} \times f_y} = \frac{W_{pl}}{W_{el}}$$

where W_{Pl} is the plastic modulus and W_{el} is the minimum elastic section modulus, I/z_1.

2 Plastic theory

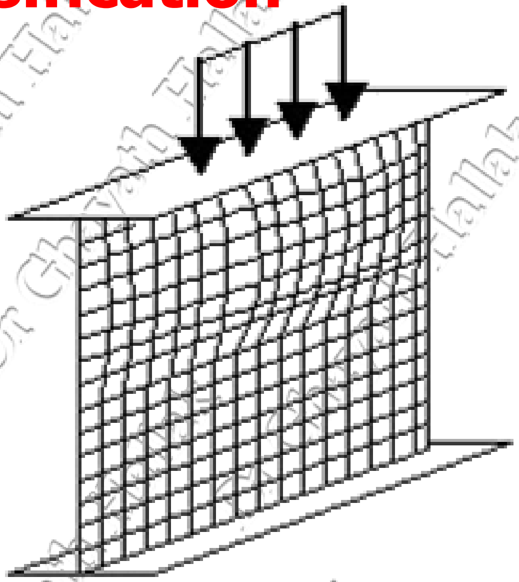
1. uniaxial bending

The plastic design resistance for bending moment given in Clause 6.2.5(1) of EN 1993-1-1,

$$M_{pl,Rd} = \frac{W_{pl}\, f_y}{\gamma_{M0}}$$

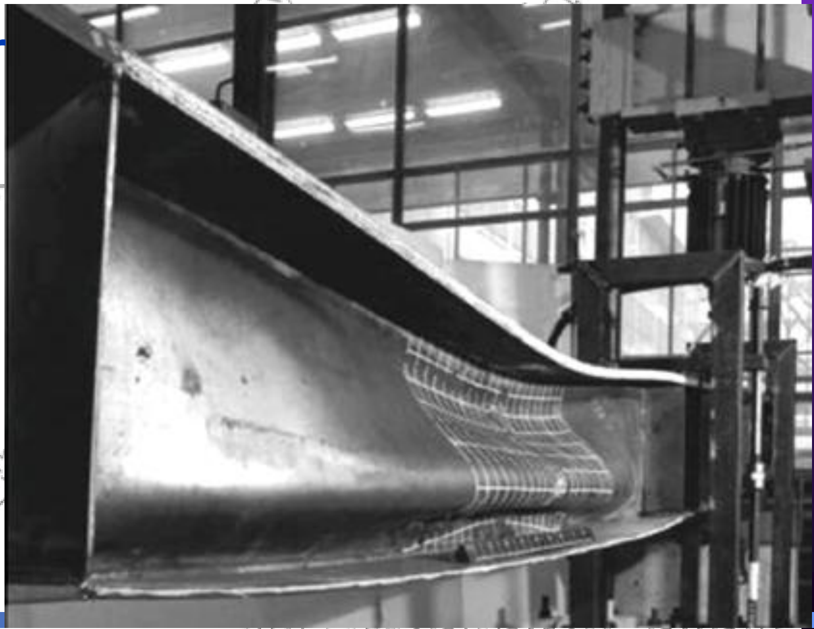
Local buckling and cross-section classification

1 Introduction



When the cross section of a steel shape is subjected to large compressive stresses, the thin plates that make up the cross section may buckle before the full strength of the member is attained if the thin plates are too slender.

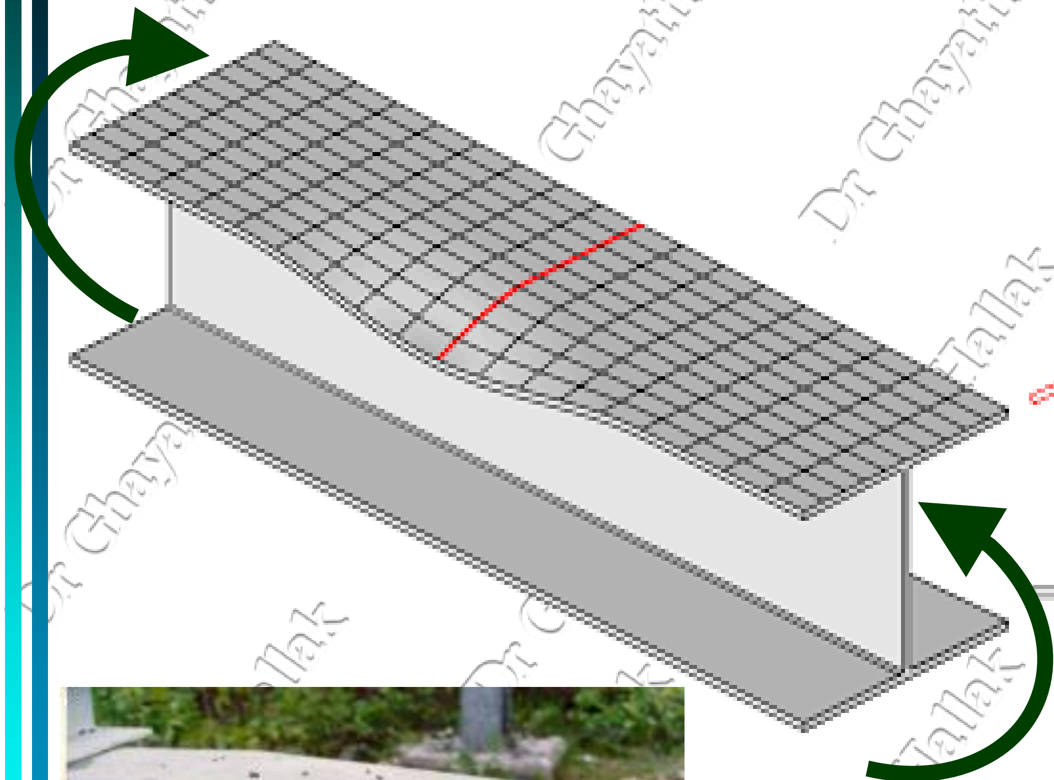
When a cross sectional element fails in buckling, then the member capacity is reached.

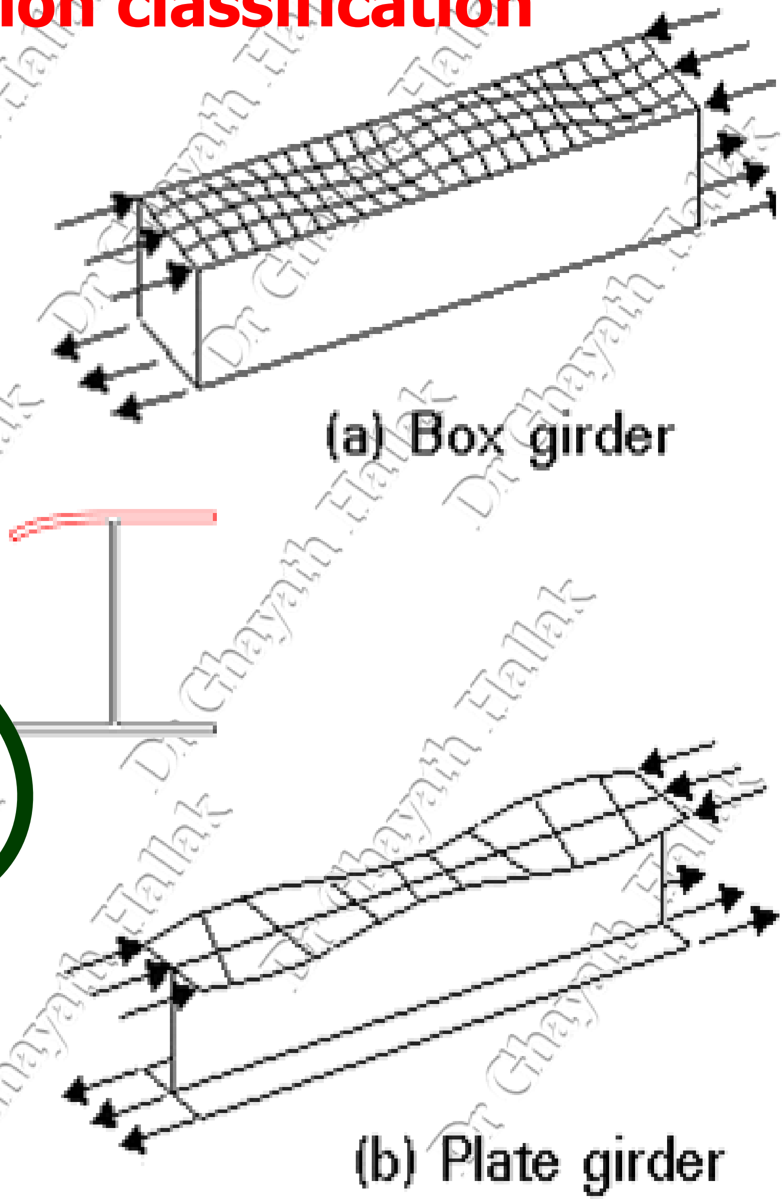


Consequently, local buckling becomes a limit state for the strength of steel shapes subjected to compressive stress.(induced by bending or axial Forces)

Local buckling and cross-section classification

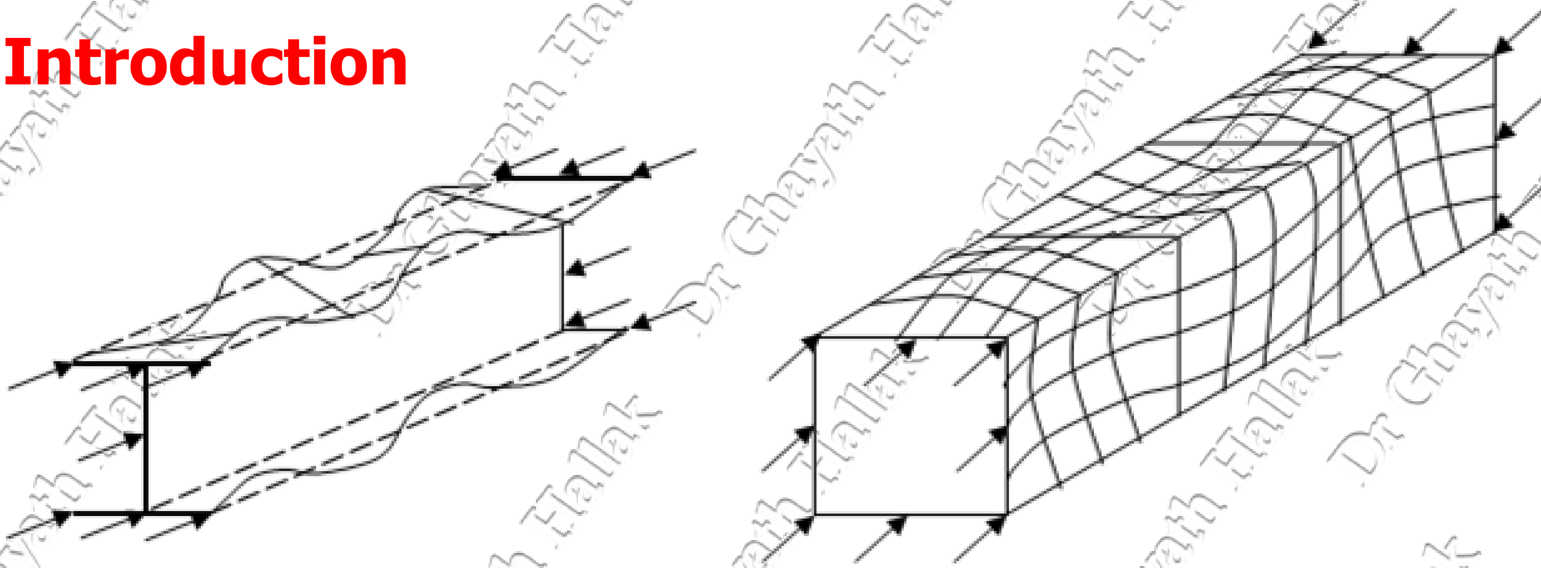
1 Introduction





Local buckling and cross-section classification

1 Introduction



- From all Previous examples, Considerable deformation of the cross-section is evident with the flanges being displaced out of their original flat shape. The web, on the other hand, appears to be comparatively undeformed.
- The buckling has therefore been confined to certain plate elements and has not resulted in any overall lateral deformation of the member (i.e. its centroidal axis has not deflected).

Local buckling and cross-section classification

1 Introduction

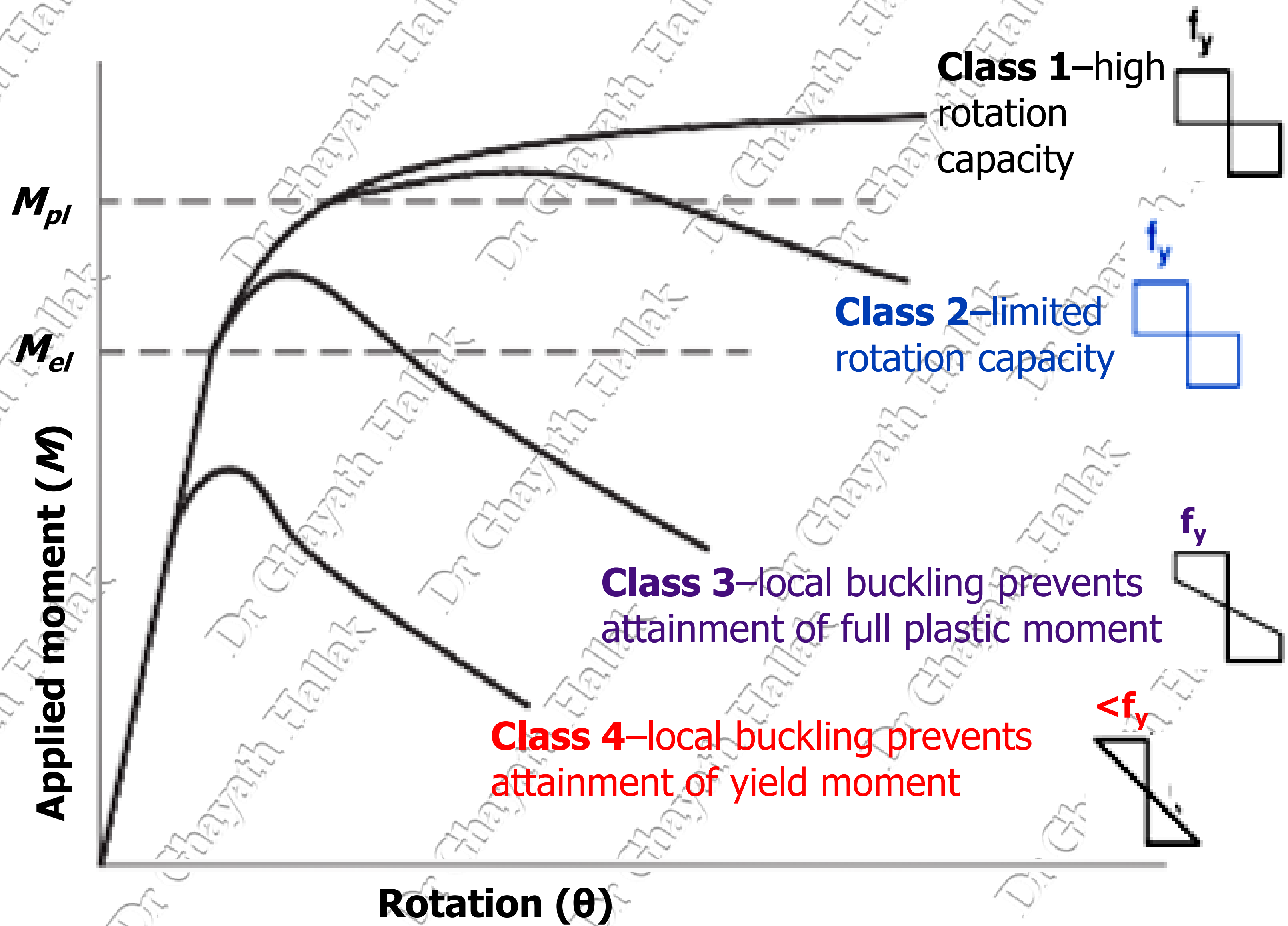
- ❑- Local buckling has the effect of reducing the load carrying capacity of columns and beams due to the reduction in stiffness and strength of the locally buckled plate elements. Therefore it is desirable to avoid local buckling before yielding of the member.
- ❑- It is useful to classify sections based on their tendency to buckle locally before overall failure of the member takes place.
- ❑- However, it should be remembered that local buckling does not always spell disaster. Local buckling involves distortion of the cross-section. There is no shift in the position of the cross-section as a whole as in global or overall buckling

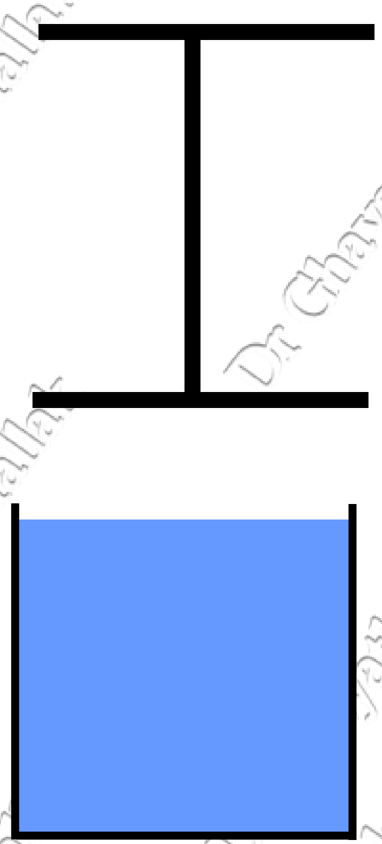
Local buckling and cross-section classification

1 Introduction

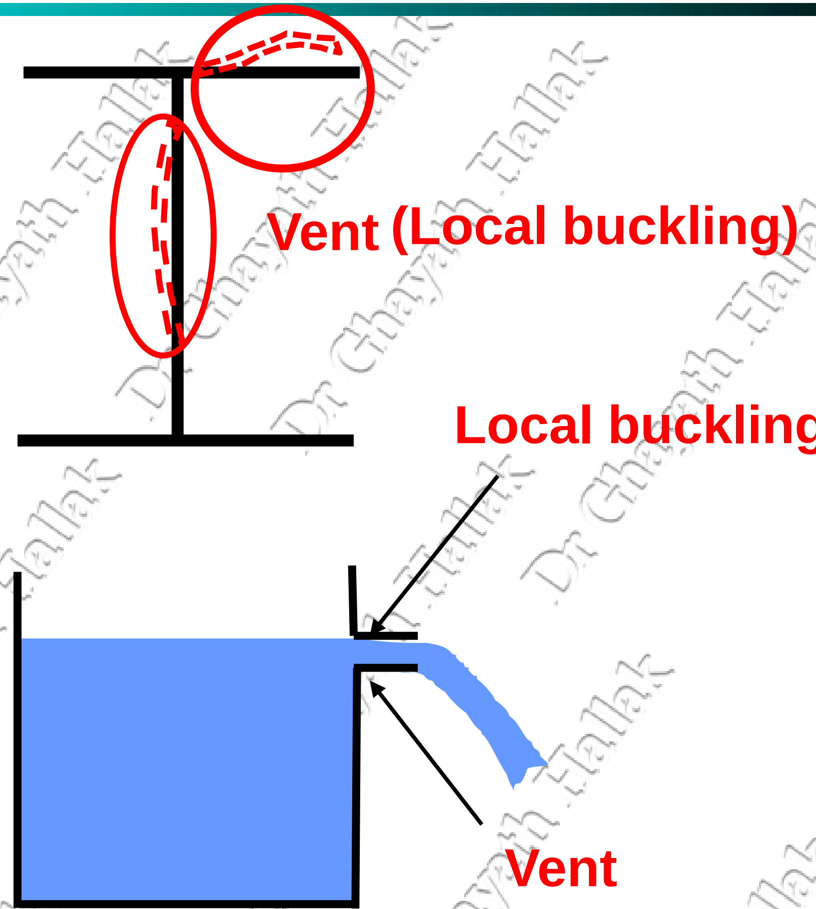
- ❑- Whether in the elastic or inelastic material range, cross-sectional resistance and rotation capacity are limited by the effects of local buckling.
- ❑- Eurocode 3 accounts for the effects of local buckling through cross-section classification.
- ❑- The factors that affect local buckling (and therefore the cross-section classification) are:
  - Width/thickness ratios of plate components
  - Element support conditions (outstand or internal flanges, internal web)
  - Material strength, f_y
  - Fabrication process (welded or hot rolled section- NO Difference in EC3)
  - Applied stress system .(stress distribution α, Ψ)

cross-section classification EN 1993-1-1 Clause 5.5

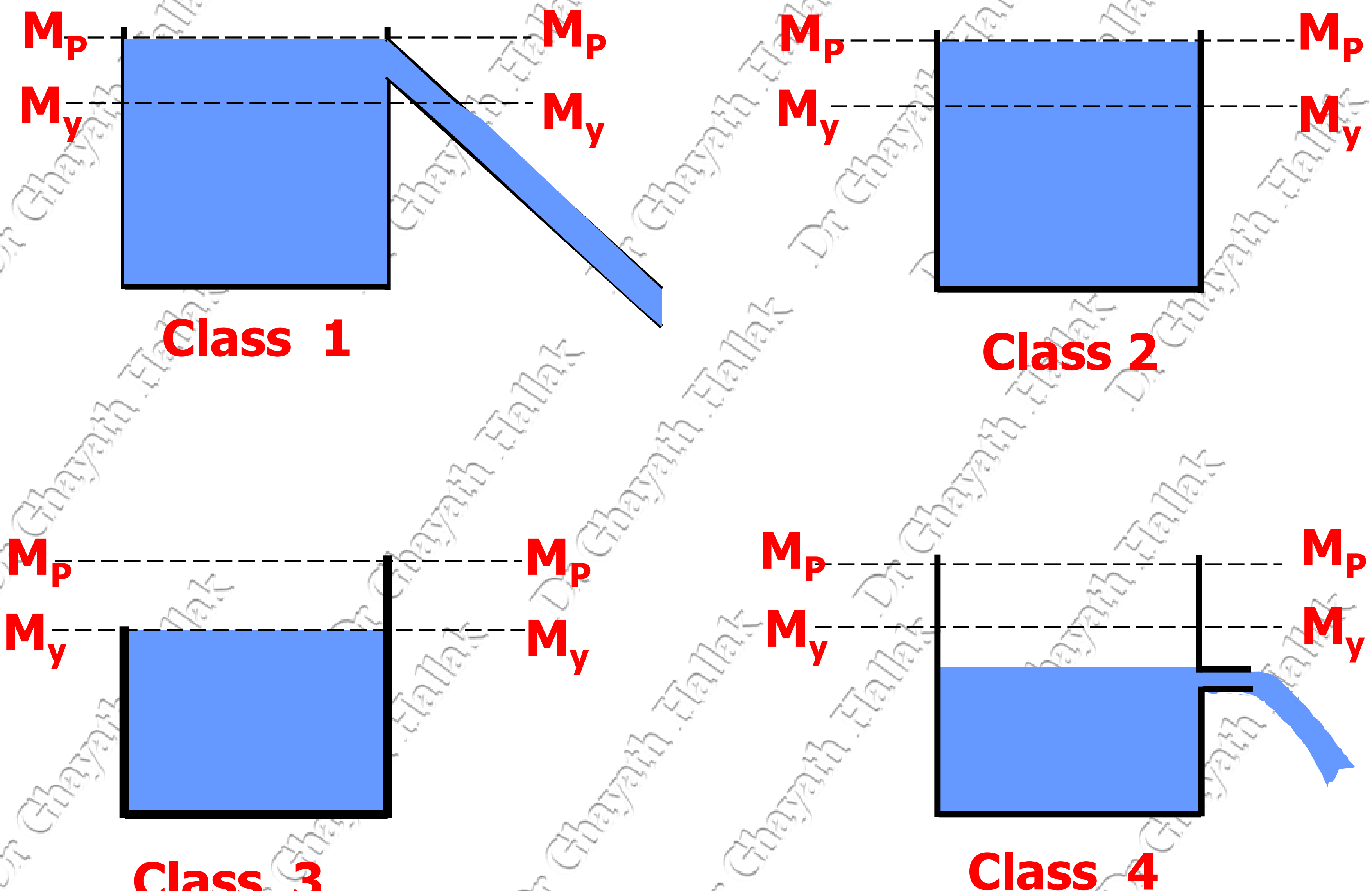




Full Section Capacity



Full section capacity
eroded by local Buckling



Mp
My
Mp
My
Class 1
Mp
My
Mp
My
Class 2
Mp
My
Mp
My
Class 3
Mp
My
Mp
My
Class 4

cross-section classification EN 1993-1-1 Clause 5.5

The EN 1993 definitions of the four beam cross sections are classified as follows in accordance with their behaviour in bending

Class 1 cross sections are those that can form a plastic hinge with rotation capacity required from plastic analysis without reduction of the resistance.

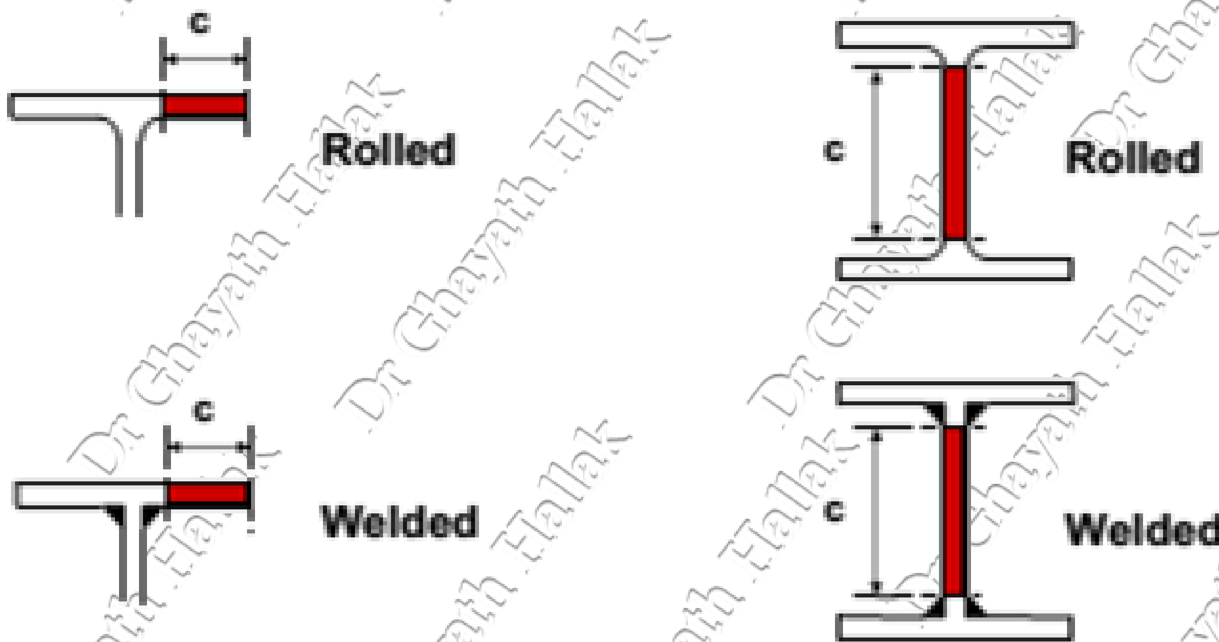
Class 2 cross sections are those that can develop their plastic moment resistance but have limited rotation capacity because of local buckling.

Class 3 cross sections are those in which the elastically calculated stress in the extreme compression fibre of the steel member assuming an elastic distribution of stresses can reach the yield strength, but local buckling is liable to prevent development of the plastic moment resistance.

cross-section classification EN 1993-1-1 Clause 5.5

Class 4 cross sections are those in which local buckling will occur before the attainment of yield stress in one or more parts of the cross section.

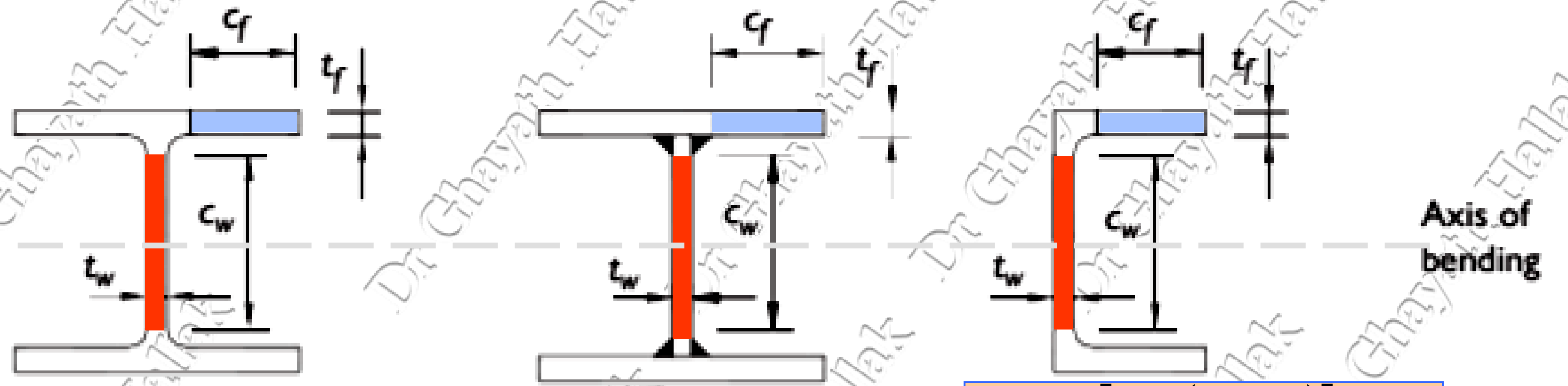
Definition of compressed widths – flat widths:



Outside elements attached on one edge with the other free

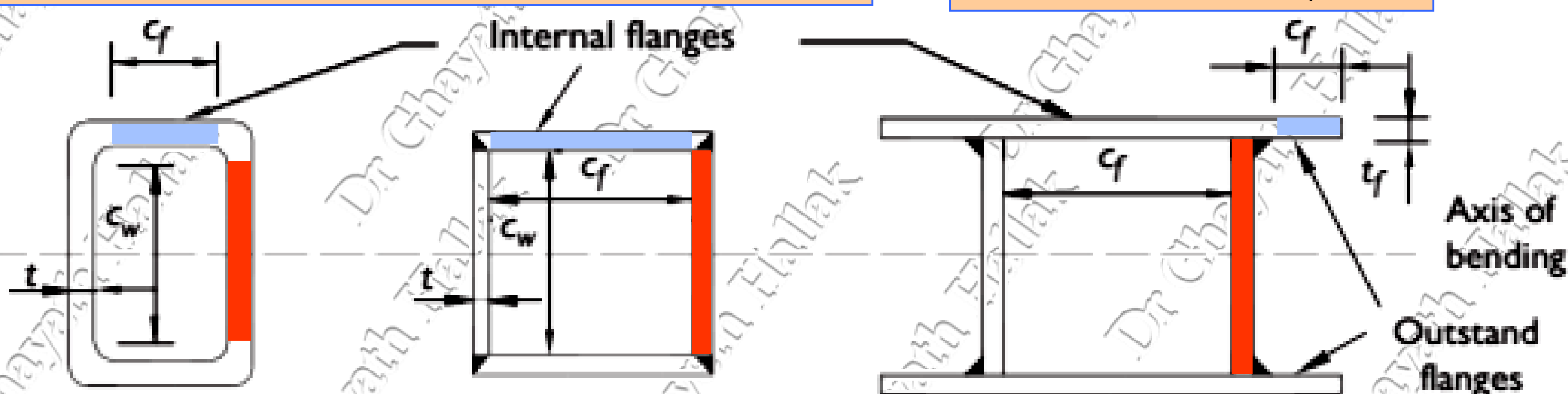
Internal elements supported on both longitudinal edges

cross-section classification EN 1993-1-1 Clause 5.5



$$C_f = \frac{1}{2}\left[b-(t_w+2r)\right],\ C_w = d = \left[h-2(t_f+r)\right]$$

$$C_f = \left[b-(t_w+r)\right]$$
$$C_w = d = \left[h-2(t_f+r)\right]$$



$$hot\ or: C_f = \left[b-3t\right]$$
$$cold\quad C_w = \left[h-3t\right]$$

cross-section classification EN 1993-1-1 Clause 5.5

- ❑- Classification is made by comparing actual width-to-thickness ratios of the plate elements with a set of limiting values, given in Table 5.2 of EN 1993-1-1.
- ❑- A plate element is Class 4 (slender) if it fails to meet the limiting values for a class 3 element.
- ❑- The **classification of the overall cross-section** is taken as the least favourable of the constituent elements (for example, a cross-section with a class 3 flange and class 1 web has an overall classification of Class 3).

Table 5.2 (sheet 1 of 3): Maximum width-to-thickness ratios

Internal compression parts

$\varepsilon = \sqrt{235/f_y}$

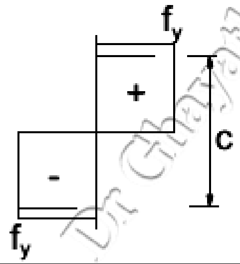
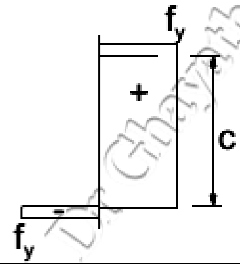
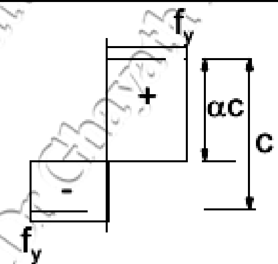
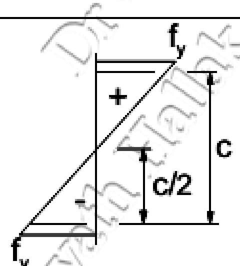
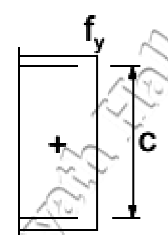
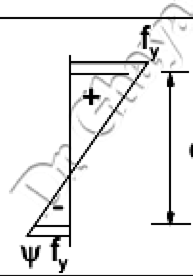
Class	Part subject to bending	Part subject to compression	Part subject to bending and compression
Stress distribution in parts (compression positive)			
1	$c/t \le 72\varepsilon$	$c/t \le 33\varepsilon$	when $\alpha > 0,5$: $c/t \le \dfrac{396\varepsilon}{13\alpha - 1}$ when $\alpha \le 0,5$: $c/t \le \dfrac{36\varepsilon}{\alpha}$
2	$c/t \le 83\varepsilon$	$c/t \le 38\varepsilon$	when $\alpha > 0,5$: $c/t \le \dfrac{456\varepsilon}{13\alpha - 1}$ when $\alpha \le 0,5$: $c/t \le \dfrac{41,5\varepsilon}{\alpha}$
Stress distribution in parts (compression positive)			
3	$c/t \le 124\varepsilon$	$c/t \le 42\varepsilon$	when $\psi > -1$: $c/t \le \dfrac{42\varepsilon}{0,67 + 0,33\psi}$ when $\psi \le -1^{*)}$: $c/t \le 62\varepsilon(1-\psi)\sqrt{(-\psi)}$

Table 5.2 (sheet 2 of 3): Maximum width-to-thickness ratios

Outstand compression flanges

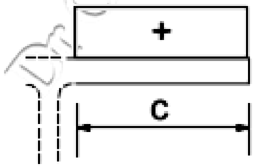
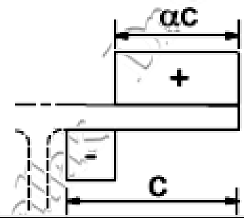
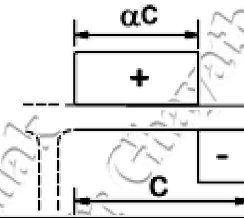
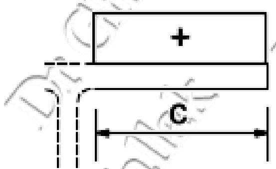
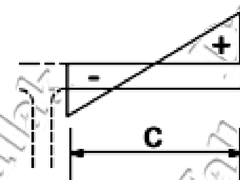
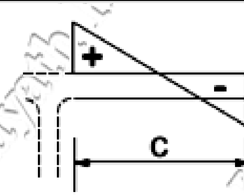
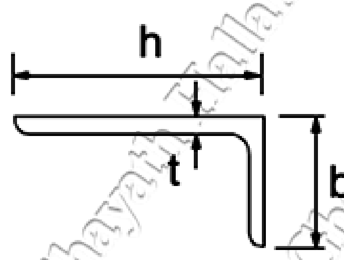
Class	Part subject to compression	Part subject to bending and compression	
		Tip in compression	Tip in tension
Stress distribution in parts (compression positive)			
1	$c/t \leq 9\varepsilon$	$c/t \leq \dfrac{9\varepsilon}{\alpha}$	$c/t \leq \dfrac{9\varepsilon}{\alpha\sqrt{\alpha}}$
2	$c/t \leq 10\varepsilon$	$c/t \leq \dfrac{10\varepsilon}{\alpha}$	$c/t \leq \dfrac{10\varepsilon}{\alpha\sqrt{\alpha}}$
Stress distribution in parts (compression positive)			
3	$c/t \leq 14\varepsilon$	$c/t \leq 21\varepsilon\sqrt{k_\sigma}$ For k_σ see EN 1993-1-5	

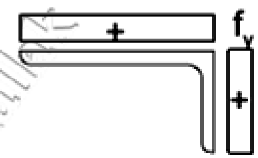
Table 5.2 (sheet 3 of 3): Maximum width-to-thickness ratios

Angles and Tubular sections

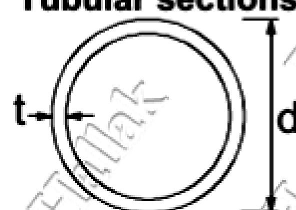
Refer also to "Outstand flanges" (see sheet 2 of 3)



Does not apply to angles in continuous contact with other components

Class	Section in compression
Stress distribution across section (compression positive)	
3	$h/t \leq 15\varepsilon : \dfrac{b+h}{2t} \leq 11{,}5\varepsilon$

Tubular sections



Class	Section in bending and/or compression
1	$d/t \leq 50\varepsilon^2$
2	$d/t \leq 70\varepsilon^2$
3	$d/t \leq 90\varepsilon^2$ **NOTE** For $d/t > 90\varepsilon^2$ see EN 1993-1-6. **See BS5950:2000**

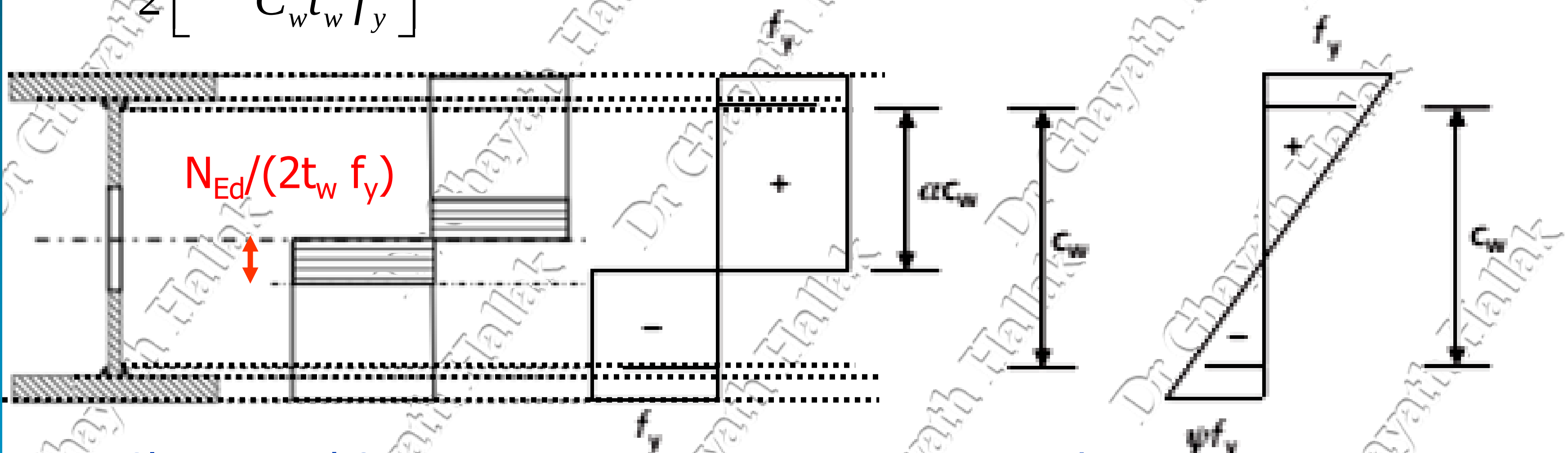
cross-section classification EN 1993-1-1 Clause 5.5

$$\alpha = \frac{1}{C_w}\left[\frac{h}{2} + \frac{1}{2}\frac{N_{Ed}}{t_w f_y} - (t_f + r)\right] = \frac{1}{2}\left[\frac{h}{C_w} + \frac{1}{C_w}\frac{N_{Ed}}{t_w f_y} - \frac{2}{C_w}(t_f + r)\right]$$

$$\alpha = \frac{1}{2}\left[\frac{h - 2(t_f + r)}{C_w} + \frac{1}{C_w}\frac{N_{Ed}}{t_w f_y}\right] =$$

$$\alpha = \frac{1}{2}\left[1 + \frac{N_{Ed}}{C_w t_w f_y}\right]$$

$$\psi = \frac{2 f_c}{f_y} - 1 = \frac{2 N_{Ed}}{A\, f_y} - 1$$



Class 1 and 2 cross-sections

Class 3 cross-sections

Definitions of α and Ψ for classification of cross-sections under combined bending and compression.

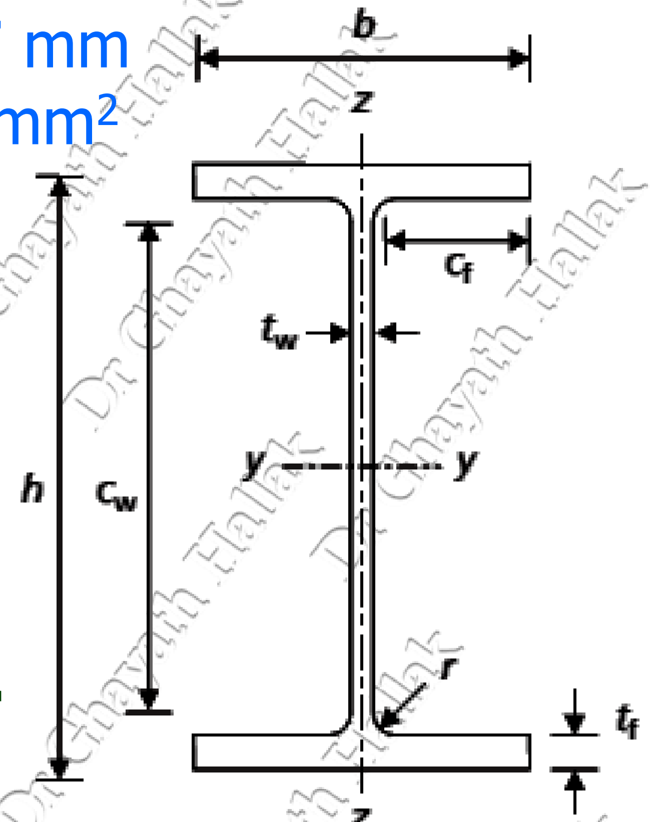
Classification influences resistance

Section classification	Compression	Bending
Class 1, 2	$N_{c,Rd}=A\ f_y/\gamma_{M0}$	$M_{c,Rd}=M_{pl}=W_{pl}\ f_y/\gamma_{M0}$
Class 3	$N_{c,Rd}=A\ f_y/\gamma_{M0}$	$M_{c,Rd}=M_{el}=W_{el}\ f_y/\gamma_{M0}$
Class 4	$N_{c,Rd}=A_{eff}\ f_y/\gamma_{M0}$	$M_{c,Rd}=W_{eff}\ f_y/\gamma_{M0}$

Example 1: cross-section classification under combined bending and compression

A member is to be designed to carry combined bending and axial load. In the presence of a major axis (y–y) bending moment and an axial force of 300 kN, determine the cross-section classification of a 406 X 178 X 54 UKB in grade S275 steel

h = 402.6 mm, b = 177.7 mm, t_w= 7.7 mm
t_f= 10.9 mm, r = 10.2 mm, A = 6900 mm^2



Since t_{max}<16mm Then f_y=275Mpa

First, classify the cross-section under the most severe loading condition of pure compression to determine whether anything is to be gained by more precise calculations.

Cross-section classification under pure compression (clause 5.5.2)

Outstand flanges (Table 5.2, sheet 2):

$$\varepsilon = \sqrt{\frac{235}{f_y}} = \sqrt{\frac{235}{275}} = 0.92$$

,C_f= (b – t_w – 2r)/2 = 74.8 mm

C_f/t_f = 74.8/10.9 = 6.86

Cross-section classification under pure compression (clause 5.5.2)

Outstand flanges (Table 5.2, sheet 2):

Limit for Class 1 flange = 9ε = 8.32

8.32 > 6.86 ; flange is Class 1

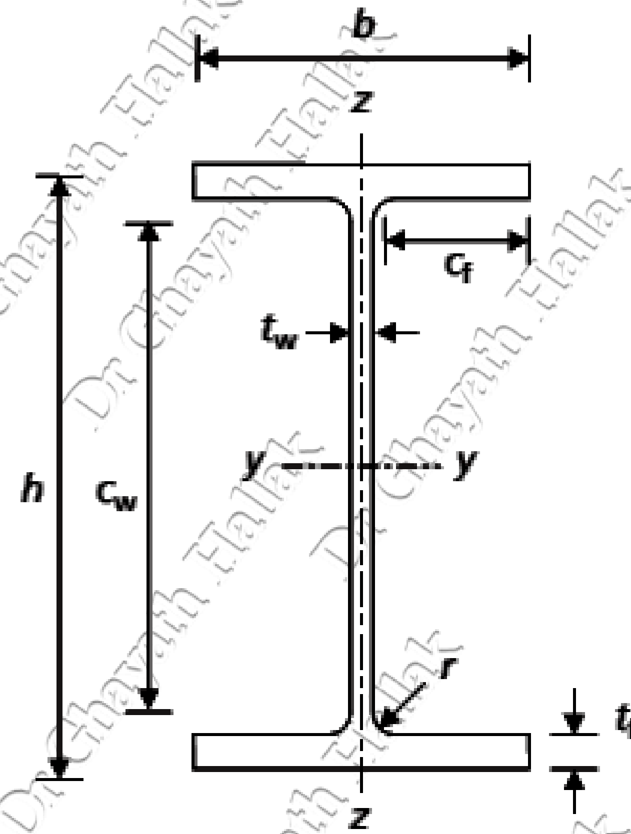
Web – internal part in compression (Table 5.2, sheet 1):

$c_w = h - 2t_f - 2r = 360.4$ mm

$c_w/t_w = 360.4/7.7 = 46.81$

Limit for Class 3 web = 42ε = 38.8

38.8 < 46.81 ; web is Class 4



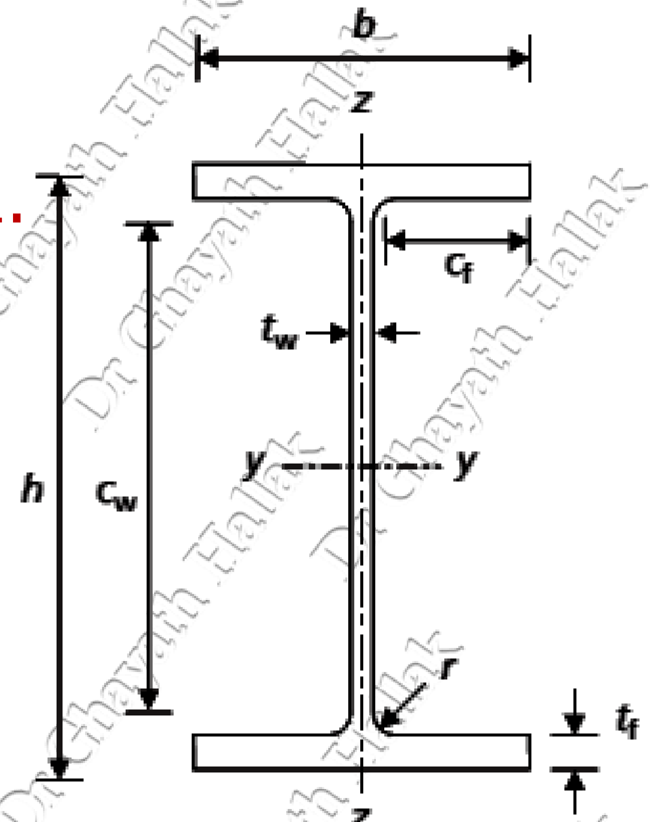
Under pure compression, the overall cross-section classification is therefore Class 4.

Calculation and material efficiency are therefore to be gained by using a more precise approach.

Cross-section classification under combined loading (clause 5.5.2)

Flange classification remains as Class 1.

Web – internal part in bending and compression (Table 5.2, sheet 1):
From Table 5.2 (sheet 1), for a Class 2 cross-section:



$$\text{when} \quad \alpha > 0.5 \quad \frac{C_w}{t_w} < \frac{456\varepsilon}{13\alpha - 1}$$

$$\text{when} \quad \alpha \le 0.5 \quad \frac{C_w}{t_w} < \frac{41.5\varepsilon}{\alpha}$$

$$\alpha = \frac{1}{2}\left[1 + \frac{N_{Ed}}{C_w t_w f_y}\right] = \frac{1}{2}\left[1 + \frac{300000}{360.4 \times 7.7 \times 275}\right] = 0.7$$

Cross-section classification under combined loading (clause 5.5.2)

Web – internal part in bending and compression (Table 5.2, sheet 1):

$$\therefore \text{limit for a Class 2 web} = \frac{456\varepsilon}{13\alpha - 1} = 52.33$$

$$52.33 > 46.81 \qquad \therefore \text{web is Class 2}$$

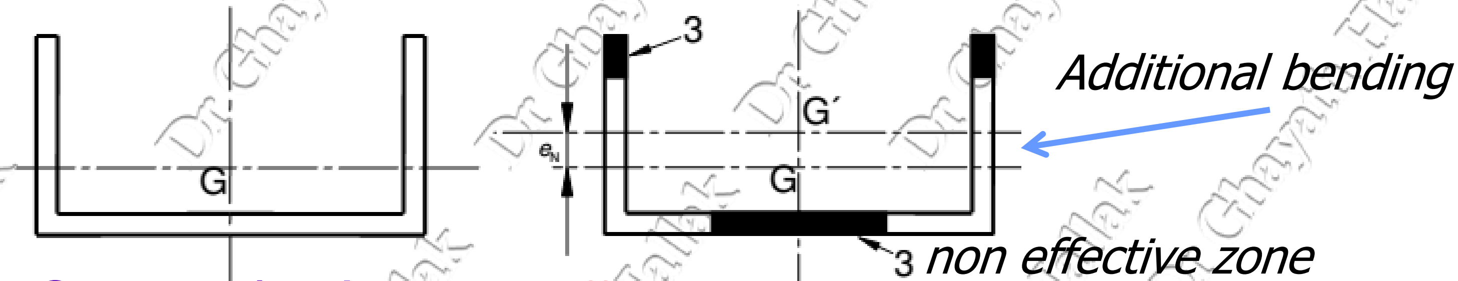
Overall cross-section classification under the combined loading is therefore Class 2.

Consider combined bending and compression

Firstly the section can be classified under the most severe loading condition of axial load only. If it is Class 4 under this condition then a more efficient classification may be obtained using a more precise calculation relating to the combined bending and axial loads.

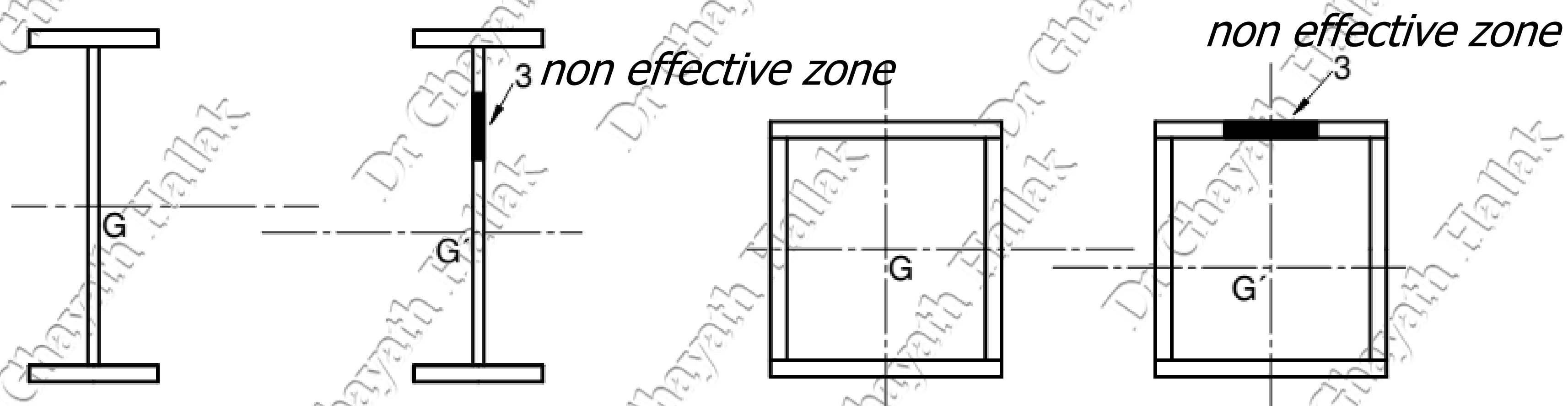
Local (plate) buckling – Class 4 cross-sections- EN 1993-1-5: 2006 § 4.3

Effective areas for Class 4 compression elements



Gross section A_c Effective section $A_{c,eff}$

Figure 4.1: Class 4 cross-sections - axial force



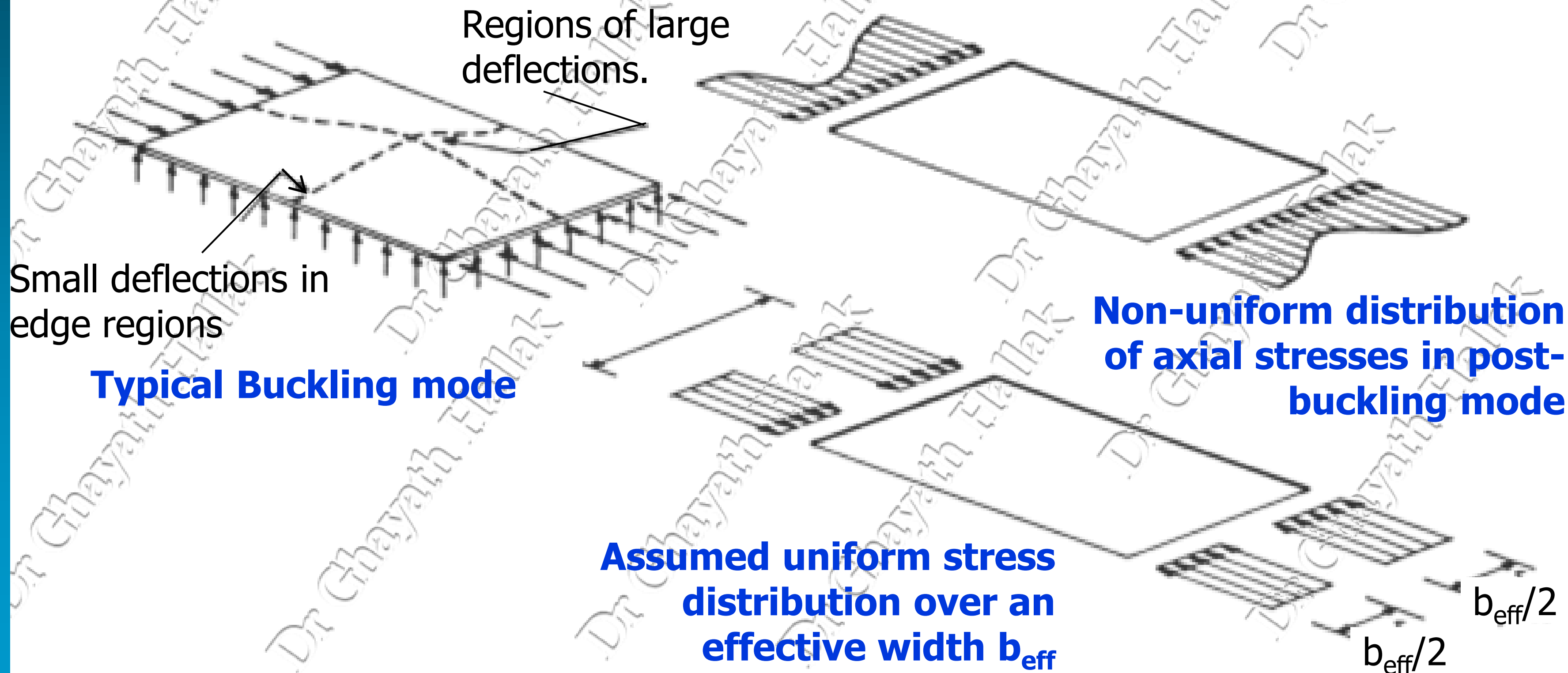
Gross section A_c Effective section $A_{c,eff}$ Gross section A_c Effective section $A_{c,eff}$

Figure 4.2: Class 4 cross-sections - bending moment

Local (plate) buckling – Class 4 cross-sections- EN 1993-1-5: 2006 § 4.3

Effective areas for Class 4 (effective width concept)

For class 4 (slender) cross-sections, reduced (effective) cross-section properties must be calculated to account explicitly for the occurrence of local buckling prior to yielding.



Effective areas – Class 4

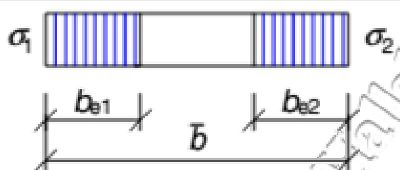
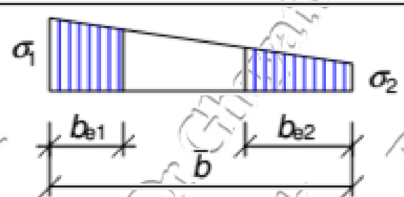
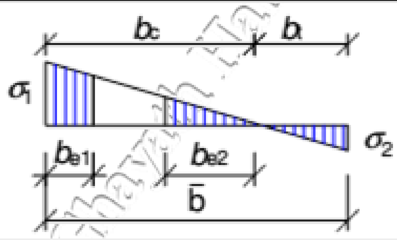
Table 4.1: Internal compression elements

$$\sigma_{cr} = \frac{k_\sigma \pi^2 E}{12(1-\upsilon^2)}\left(\frac{t}{\bar{b}}\right)^2$$

$$\rho = 1.0 \qquad for\ \bar{\lambda}_p \le 0.673$$

$$\rho = \frac{\bar{\lambda}_p - 0.055(3+\psi)}{\bar{\lambda}_p^2} \le 1.0\ for\ \bar{\lambda}_p > 0.673,\ where\ (3+\psi) \ge 0,\ \bar{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{cr}}} = \frac{\bar{b}/t}{28.4\,\varepsilon\sqrt{k_\sigma}}$$

EN 1993-1-5: 2006 § 4.3

Stress distribution (compression positive)	Effective[p] width b_{eff}
	$\underline{\psi = 1}$: $b_{eff} = \rho\ \bar{b}$ $b_{e1} = 0{,}5\ b_{eff}$ $\quad b_{e2} = 0{,}5\ b_{eff}$
	$\underline{1 > \psi \ge 0}$: $b_{eff} = \rho\ \bar{b}$ $b_{e1} = \frac{2}{5-\psi} b_{eff}$ $\quad b_{e2} = b_{eff} - b_{e1}$
	$\underline{\psi < 0}$: $b_{eff} = \rho\ b_c = \rho\ \bar{b} / (1-\psi)$ $b_{e1} = 0{,}4\ b_{eff}$ $\quad b_{e2} = 0{,}6\ b_{eff}$

$\psi = \sigma_2/\sigma_1$	1	$1 > \psi > 0$	0	$0 > \psi > -1$	-1	$-1 > \psi > -3$
Buckling factor k_σ	4,0	$8{,}2 / (1{,}05 + \psi)$	7,81	$7{,}81 - 6{,}29\psi + 9{,}78\psi^2$	23,9	$5{,}98\ (1 - \psi)^2$

Table 4.1

Effective areas – Class 4

Table 4.2: Outstand compression elements

$$\rho = 1.0 \qquad for\ \bar{\lambda}_p \leq 0.748$$

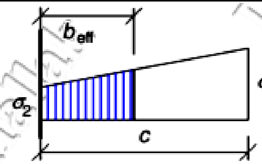
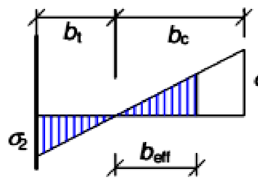
$$\rho = \frac{\bar{\lambda}_p - 0.188}{\bar{\lambda}_p^2} \leq 1.0 \qquad for\ \bar{\lambda}_p > 0.748$$

Table 4.2

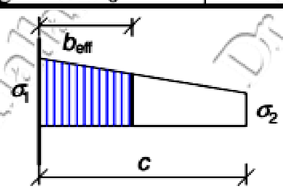
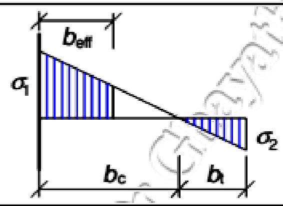
EN 1993-1-5: 2006 § 4.3

$$, \bar{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{cr}}} = \frac{\bar{b}/t}{28.4\,\varepsilon\sqrt{k_\sigma}}$$

$$\sigma_{cr} = \frac{k_\sigma \pi^2 E}{12(1-\upsilon^2)}\left(\frac{t}{\bar{b}}\right)^2$$

Stress distribution (compression positive)	Effective^p width b_{eff}
	$1 > \psi \geq 0$: $b_{eff} = \rho\, c$
	$\psi < 0$: $b_{eff} = \rho\, b_c = \rho\, c\,/\,(1-\psi)$

$\psi = \sigma_2/\sigma_1$	1	0	-1	$1 \geq \psi \geq -3$
Buckling factor k_σ	0,43	0,57	0,85	$0{,}57 - 0{,}21\psi + 0{,}07\psi^2$

Stress distribution (compression positive)	Effective^p width b_{eff}
	$1 > \psi \geq 0$: $b_{eff} = \rho\, c$
	$\psi < 0$: $b_{eff} = \rho\, b_c = \rho\, c\,/\,(1-\psi)$

$\psi = \sigma_2/\sigma_1$	1	$1 > \psi > 0$	0	$0 > \psi > -1$	-1
Buckling factor k_σ	0,43	$0{,}578\,/\,(\psi + 0{,}34)$	1,70	$1{,}7 - 5\psi + 17{,}1\psi^2$	23,8

Effective areas – Class 4 -EN 1993-1-5: 2006 § 4.3

Notes 0n Table 4.1:

$\bar{b}$ is the appropriate width to be taken as follows (for definitions, see Table 5.2 of EN 1993-1-1)

$b_w = c_w$ for webs; (clear width between welds)

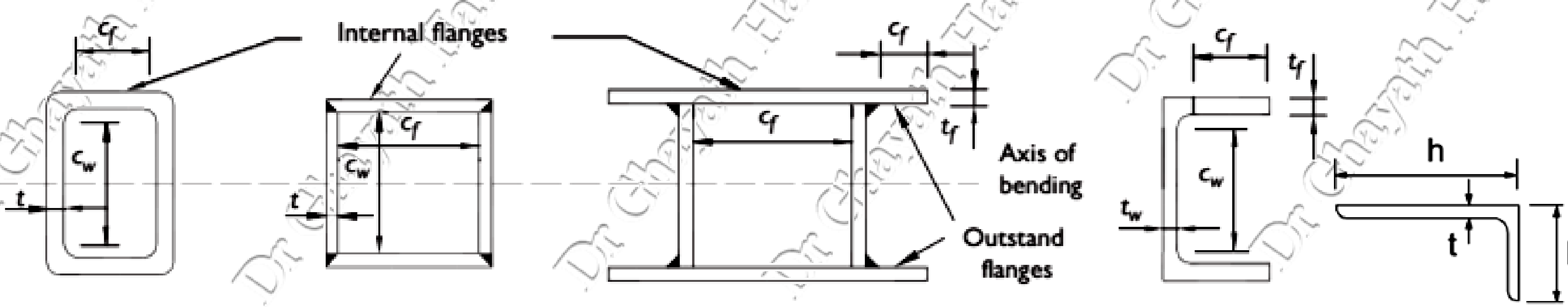
$b = c_f$ for internal flange elements (except RHS);

b - 3 t for flanges of RHS;

c_f for outstand flanges;

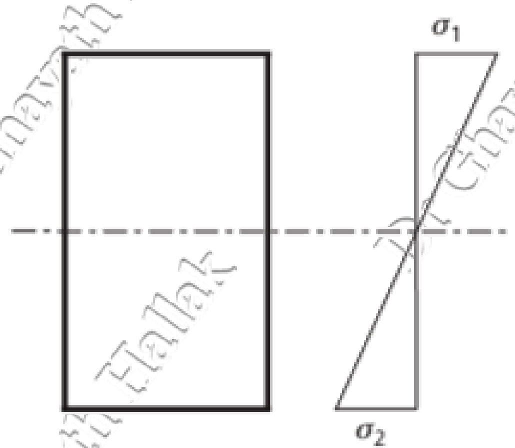
h for equal-leg angles;

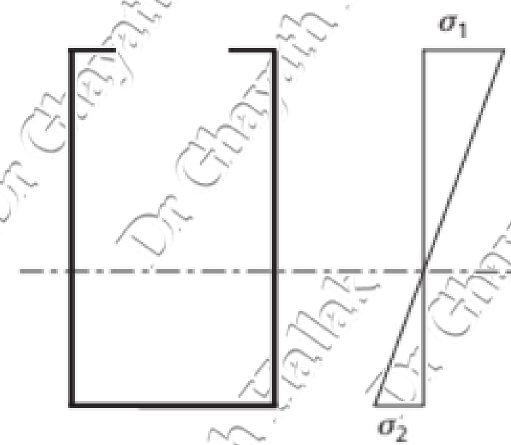
h for unequal-leg angles;



Effective areas – Class 4 -EN 1993-1-5: 2006 § 4.3

Notes 0n Table 4.1:



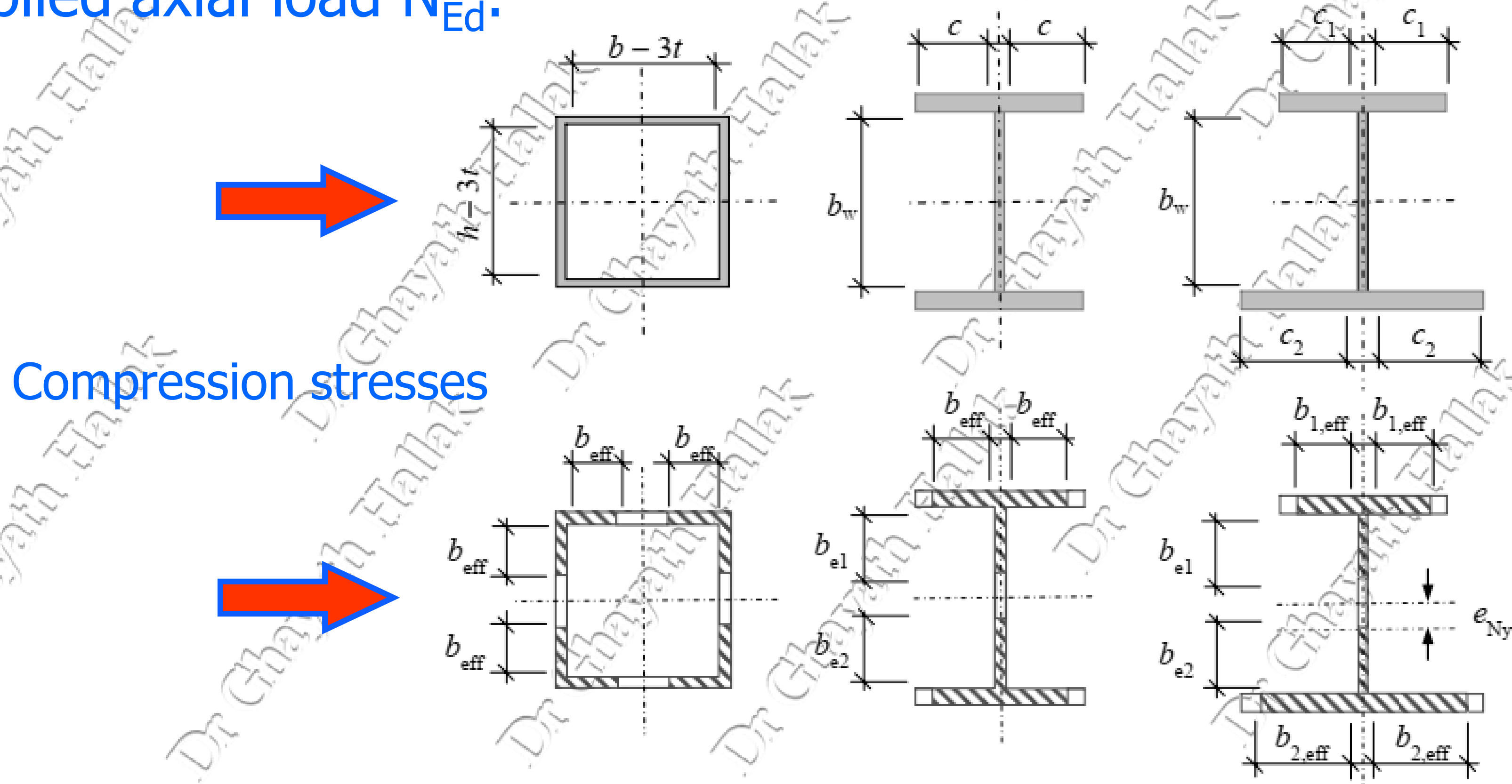


- for **flange elements**, the stress ratio Ψ should be based on the properties of the **gross** cross-section.

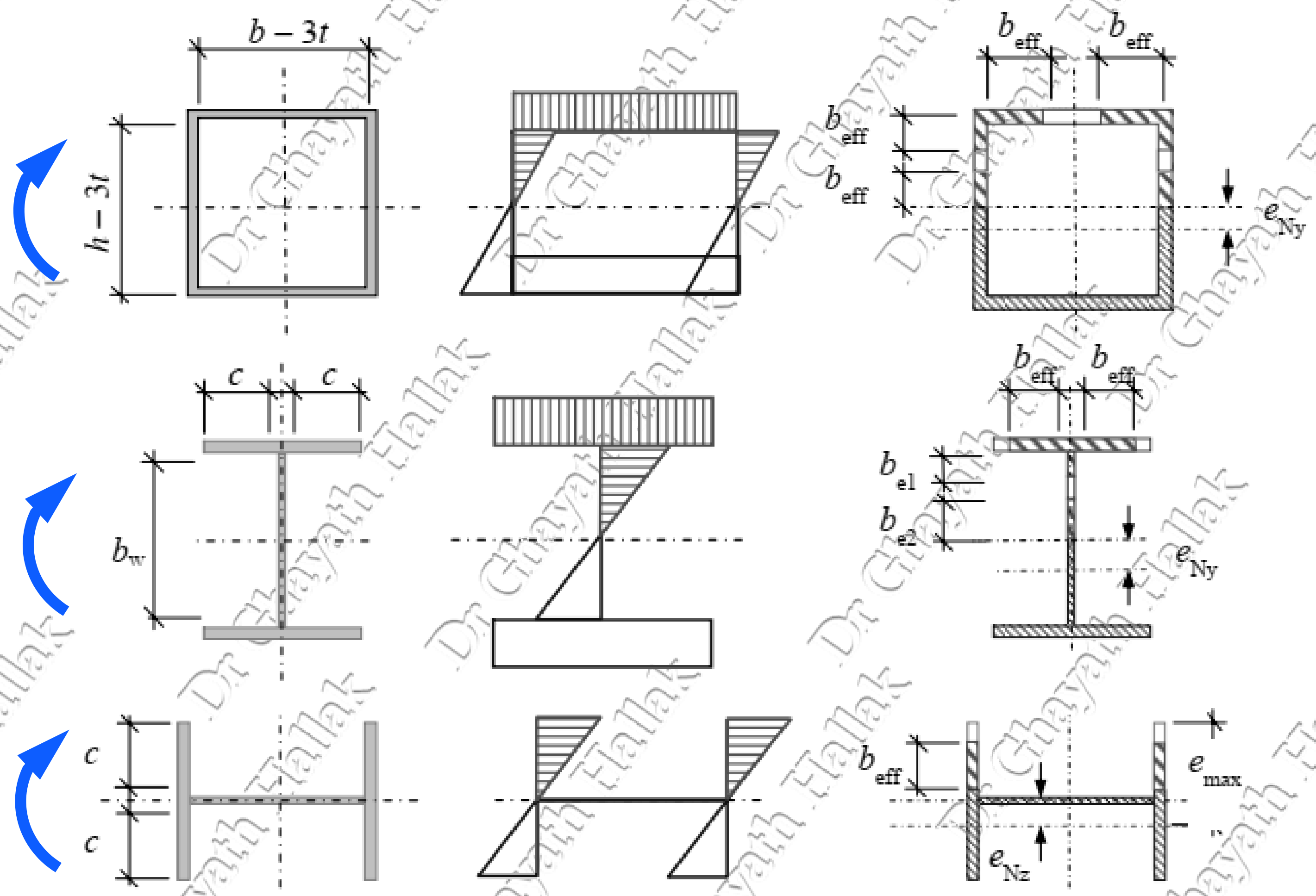
- for **web elements**, the stress ratio Ψ should be found using a stress distribution obtained with the **effective area of the compression flange** and the **gross area of the web**

Effective areas – Class 4 -EN 1993-1-5: 2006 § 4.3

In most cases, the introduction of non-effective zones in a cross-section, or part of a cross-section, will shift the position of the neutral axis for the 'effective cross-section'. This introduces an additional bending moment due to the eccentricity of the applied axial load N_{Ed}.



Effective areas – Class 4 -EN 1993-1-5: 2006 § 4.3

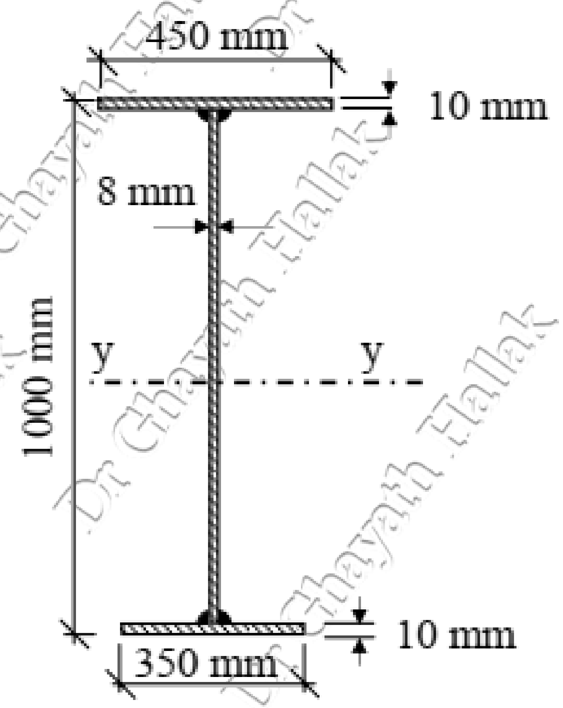


Bending stresses

Effective areas – Class 4 -EN 1993-1-5: 2006 § 4.3

Example : Effective Cross-section Properties

Using the design data given for the welded I-section indicated in the Figure shown, determine the section classification, and
(i) the effective cross-sectional area when the section is subject to compression,
(ii) the effective elastic section modulus when the section is subject to bending.



Design data:

Steel grade = S275
Assume 6 mm fillet welds
Assume that any shear lag effects, see EN 1993-1-5: Clause 3.1(1), are negligible

Solution:

Gross cross-section properties
Cross-sectional area $A = \sum A_i = (450 \times 10) + (980 \times 8) + (350 \times 10) = 15840 \text{ mm}^2$

Effective areas – Class 4 -EN 1993-1-5: 2006 § 4.3

Example : Effective Cross-section Properties

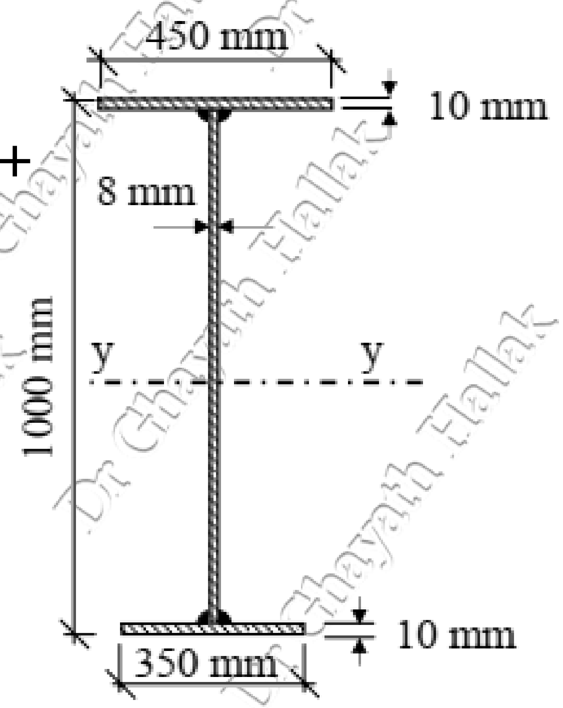
Solution:

Distance to the centroid from the bottom flange

$z_c = \sum A_i z_i / \sum A_i$ = [(450 × 10 × 995) + (980 × 8 × 500) + (350 × 10 × 5)]/15840= 531,25 mm

Second moment of inertia with respect to the y-y axis

I_{yy} = (450x10^3)/12+450x10x(1000-5-531.25)^2 + 980^{3}x8/12+980x8x(531.25-500)^2 + (350x10^3)/12+ 350x10x(531.25-5)^2= 2572.26 × 10^6 mm^4



EN 10025-2:2004

S275 steel: For $t \leq$ 16 mm f_y = 275 MPa

$$\varepsilon = \sqrt{\frac{235}{f_y}} = \sqrt{\frac{235}{275}} = 0.92$$

Effective areas – Class 4 -EN 1993-1-5: 2006 § 4.3

Example : Effective Cross-section Properties

Solution:

(i) Consider compression

Web: internal compression part- Table 5.2(1)-

$c = [1000 - 20 - (2 \times 6)] = 968$ mm $c/t_w = 968/8 = 121$

$42\varepsilon = (42 \times 0.92) = 38.64$ $c/t_w > 42\varepsilon$ ∴ The web is Class 4

Upper flange: outstand compression flanges - Table 5.2(2)-

$c = [450 - 8 - (2 \times 6)]/2 = 215$ mm

$c/t_f = 215/10 = 21.5$

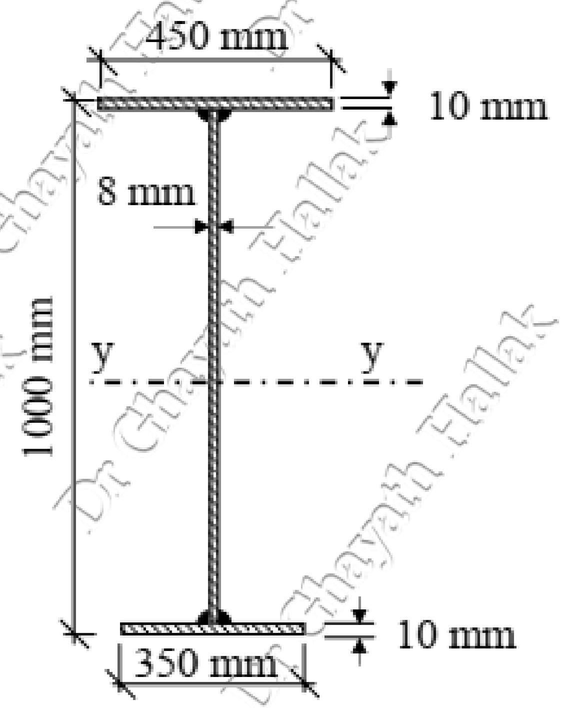
Lower flange: outstand compression flanges

$c = [350 - 8 - (2 \times 6)]/2 = 165$ mm

$c/t_f = 165/10 = 16.5$

$14\varepsilon = (14 \times 0.92) = 12.55$ $c/t_f > 14\varepsilon$ Both flanges are Class 4

∴ **Section is Class 4**



Effective areas – Class 4 -EN 1993-1-5: 2006 § 4.3

Example : Effective Cross-section Properties

(i) Consider compression

Effective area, A_{eff} (EN 1993-1-5:2006 Clauses 4.3 & 4.4)

Assume the cross-section is subject only to stresses due to uniform axial compression

$A_{c,eff} = \rho A_c$, $\bar{\lambda}_p = \sqrt{\dfrac{f_y}{\sigma_{cr}}} = \dfrac{\bar{b}/t}{28.4\varepsilon\sqrt{k_\sigma}}$ plate slenderness

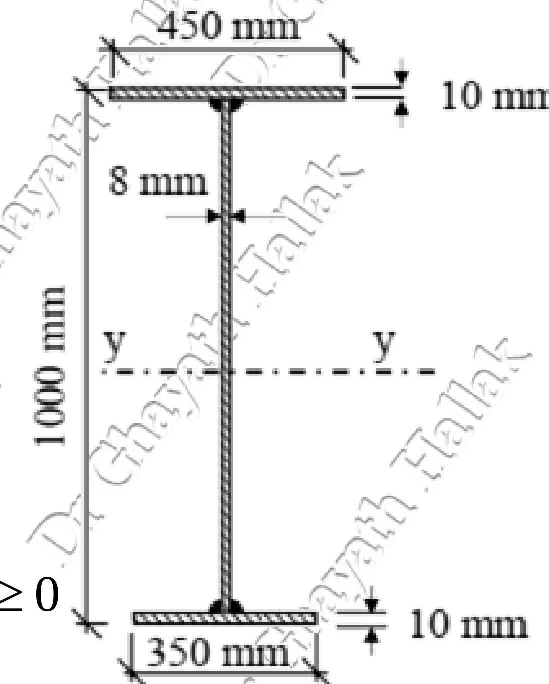


Table 4.1 **For internal compression elements** (WEB) with uniform compression

$\psi = \sigma_2/\sigma_1 = 1{,}0$ and $k_\sigma = 4{,}0$

$$\therefore \bar{\lambda}_p == \frac{121}{28.4 \times 0.92 \times \sqrt{4.0}} = 2.315$$

Reduction factor

$$\rho = 1.0 \qquad for\ \bar{\lambda}_p \le 0.673$$

$$\rho = \frac{\bar{\lambda}_p - 0.055(3+\psi)}{\bar{\lambda}_p^2} \le 1.0\ for\ \bar{\lambda}_p > 0.673,\ where\ (3+\psi) \ge 0$$

$$\rho = \frac{2.315 - 0.055(3+1)}{2.315^2} = 0.391 \le 1.0$$

Effective areas – Class 4 -EN 1993-1-5: 2006 § 4.3

Example : Effective Cross-section Properties

(i) Consider compression

b_{eff} = (0,391 × 968) = 378,49 mm

50% allocated equally from both ends (i.e. 189,25 from the welds)

$A_{c,eff,w}$ = (378,49 + 6,0 + 6,0) × 8,0 = 3123,9 mm²

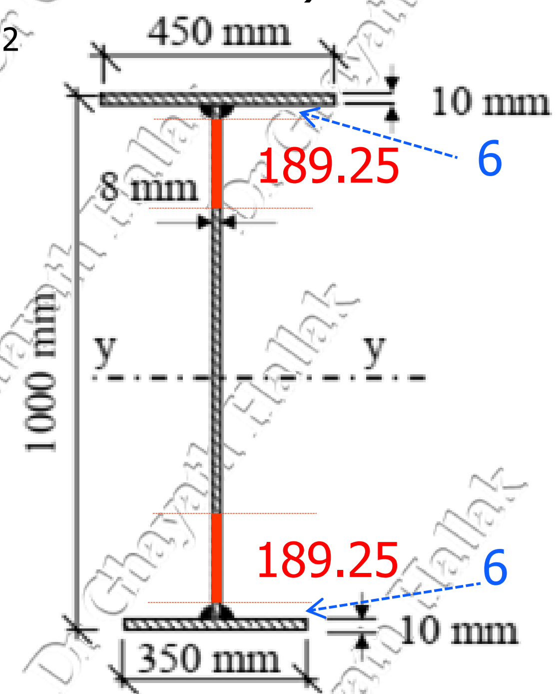


Table 4.2 For **outstand compression elements** with uniform compression

$\psi = \sigma_2/\sigma_1 = 1{,}0$ and $k_\sigma = 0.43$

$$\therefore \bar{\lambda}_p = \frac{\bar{b}/t}{28.4 \times 0.92 \times \sqrt{0.43}} = 0.058\,\bar{b}/t$$

Reduction factor

$$\rho = 1.0 \qquad for\ \bar{\lambda}_p \le 0.748$$

$$\rho = \frac{\bar{\lambda}_p - 0.188}{\bar{\lambda}_p^2} \le 1.0 \qquad for\ \bar{\lambda}_p > 0.748$$

For the upper flange

$$\therefore \bar{\lambda}_p = 0.058\,\bar{b}/t = 0.058 \times 21.5 = 1.247 \quad \Rightarrow \rho = \frac{1.247 - 0.188}{1.247^2} = 0.681 \langle 1.0$$

Effective areas – Class 4 -EN 1993-1-5: 2006 § 4.3

Example : Effective Cross-section Properties

(i) Consider compression

b_{eff} = (0.681 × 215) = 146.42 mm

$A_{c,eff,tf}$ = 2 × [(146,42 + 6,0 + 4,0) × 10,0] = 3128,4 mm²

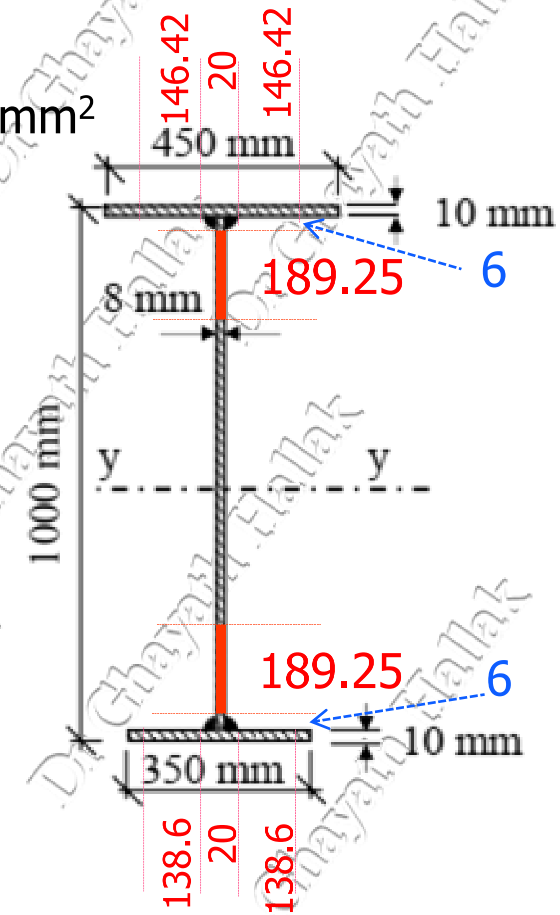
For the lower flange:

$$\therefore \bar{\lambda}_p = 0.058\,\bar{b}/t = 0.058 \times 16.5 = 0.957$$

$$\Rightarrow \rho = \frac{0.957 - 0.188}{0.957^2} = 0.84 \langle 1.0$$

b_{eff} = (0.84 × 165) = 138.6 mm

$A_{c,eff,bf}$ = 2 × (138,6 + 6,0 + 4) × 10,0 = 2972,0 mm²



Effective areas – Class 4 -EN 1993-1-5: 2006 § 4.3

Example : Effective Cross-section Properties

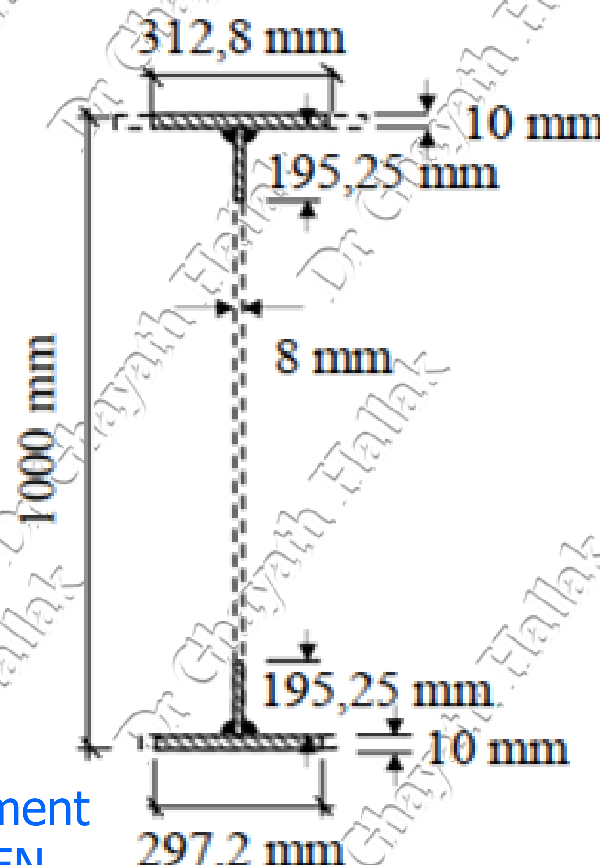
(i) Consider compression

Effective section properties:

A_{eff} = (3123,9 + 3128,4 + 2972) = 9224,3 mm²

Location of centroid

Z_{eff} = [(3128,4 × 995) + (195.25 x 8× (1000-10-195.25/2))+ (195.25 x 8 × (195.25/2+10)) + (2972 × 5)]/9224,2=508.4mm



Shift of the centroid:

e_{Ny} = 531,25 - 508,40 = 22,85 mm

This shift results in an additional bending moment which should be added to the primary bending moment when carrying out verifications in accordance with EN 1993-1-1: Clause 6.2.2.3(4), i.e. $\Delta M_{Ed} = N_{Ed} e_N$

Effective areas – Class 4 -EN 1993-1-5: 2006 § 4.3

Example : Effective Cross-section Properties

(ii) Consider bending

Upper flange: outstand compression flanges - Table 5.2(2)-

$c = [450 - 8 - (2 \times 6)]/2 = 215$ mm

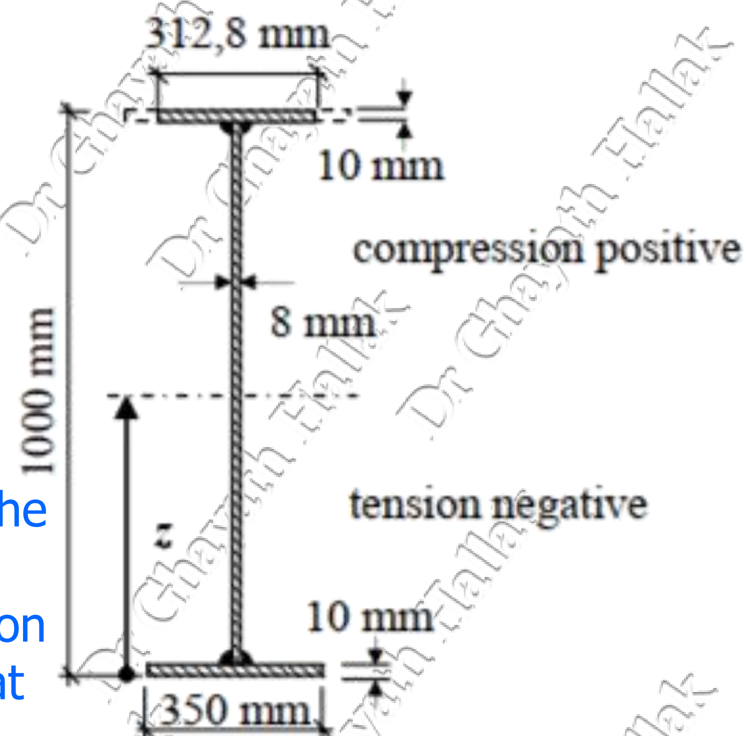
$c/t_f = 215/10 = 21.5$

$14\varepsilon = (14 \times 0.92) = 12.55$ $c/t_f > 14\varepsilon$

compression flange is Class 4

Web: internal compression part

The classification of the web is dependent on the stress ratio ψ, see EN 1993-1-1: Table 5.2(1). Since the flanges provide the largest contribution to the bending stiffness, it is recommended that the compression flange is first reduced before completing the stress distribution over the depth of the section.



The cross-section considered when assessing the stress ratio from EN 1993-1-3: Table 5.2 is shown in the Figure. The effective width of the top flange is equal to 312,8 mm as before.

Effective areas – Class 4 -EN 1993-1-5: 2006 § 4.3

Example : Effective Cross-section Properties

(ii) Consider bending

Cross-sectional area:

A = (312,8 × 10) + (980 × 8) + (350 × 10) = 14468 mm^2

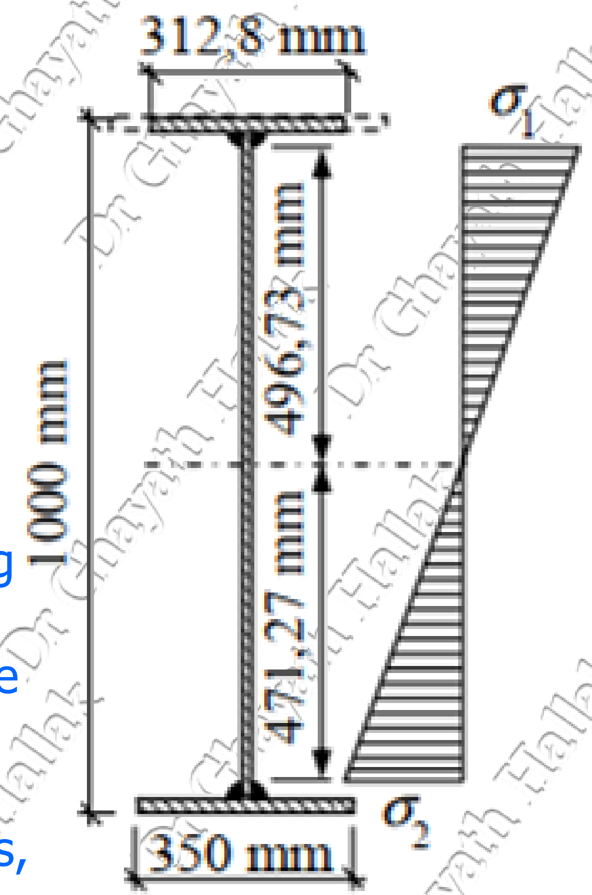
Distance to the centroid from the bottom flange:

z = [(312,8 × 10 × 995) + (980 × 8 × 500) + (350 × 10 × 5)]/14468 = 487,27 mm

The cross-section to be considered when assessing the stress ratio is as shown in the Figure. The stresses σ1 and σ2 are proportional to the distance from the centroid.

The stress ratio is based on the values to the extreme fibres of the web plate between the welds, i.e

ψ = - 471,27/496,73 = - 0,949 > - 1,0



Effective areas – Class 4 -EN 1993-1-5: 2006 § 4.3

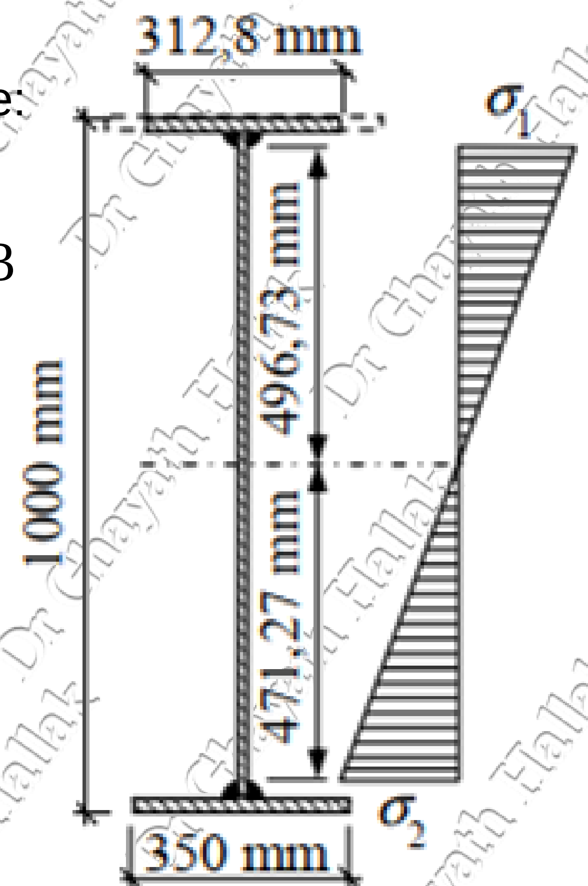
Example : Effective Cross-section Properties

(ii) Consider bending

EN 1993-1-1:2005: Table 5.2: Class 3 limiting value:

$$\frac{C}{t} = \frac{42\varepsilon}{0.67+0.33\psi} = \frac{42\times 0.92}{0.67-0.33\times 0.949} = 108.3$$

$c/t_w = 121 > 108.3 \therefore$ The web is Class 4



Section is Class 4

Effective section modulus

The effective section modulus (W_{eff}) is determined assuming that the cross-section is subject only to bending stresses or combined bending and compression. Assuming that shear lag is negligible, the tension flange is fully effective and the web is subject to combined bending and compression.

Using the stress ratio ψ = - 0,949 as a first approximation, the buckling coefficient k_σ can be obtained from EN 1993-1-5: Table 4.1.

Effective areas – Class 4 -EN 1993-1-5: 2006 § 4.3

Example : Effective Cross-section Properties

(ii) Consider bending

Buckling factor k_σ = 7,81 - 6,29ψ + 9,78ψ ^{2} where ψ ≈ - 0,949

k_σ = 7,81 + (6,29 × 0,949) + (9,78 × 0,949²) = 22,59

$$\therefore \bar{\lambda}_p = \frac{121}{28.4 \times 0.92 \times \sqrt{22.59}} = 0.974 > 0.673$$

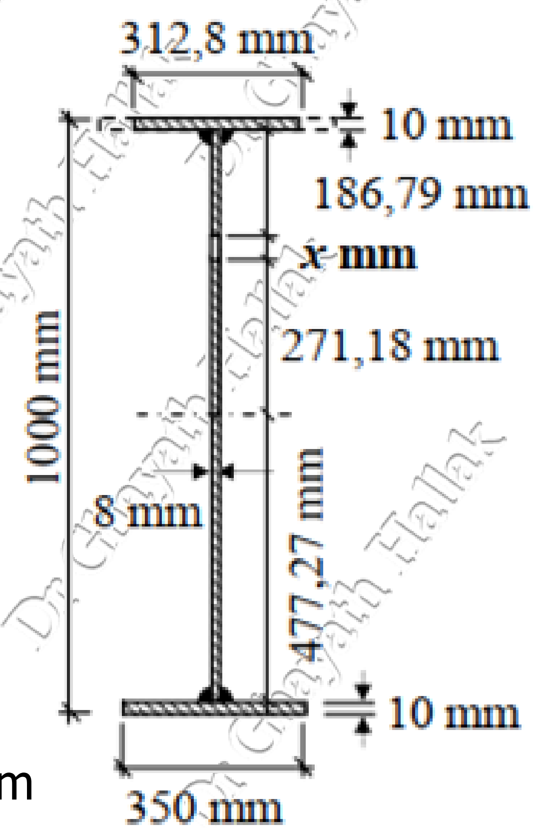
$$\rho = \frac{0.974 - 0.055(3 - 0.949)}{0.974^2} = 0.908 \leq 1.0$$

b_{eff} = ρb_c = ρ b /(1- ψ) = (0,908 × 968,0)/(1+ 0,949)
= 451,97 mm

The b_{eff} value (451,97 mm) is allocated to the compression zone in accordance with EN 1993-1-5: Table 4.1, i.e

b_{e1} = 0,4b_{eff} = (0,4 × 451,97) = 180,79 mm
b_{e2} = 0,6b_{eff} = (0,6 × 451,97) = 271,18 mm

x = 1000 - (20 + 186,79 + 271,18 + 477,27) = 44,76 mm



Effective areas – Class 4 -EN 1993-1-5: 2006 § 4.3

Example : Effective Cross-section Properties

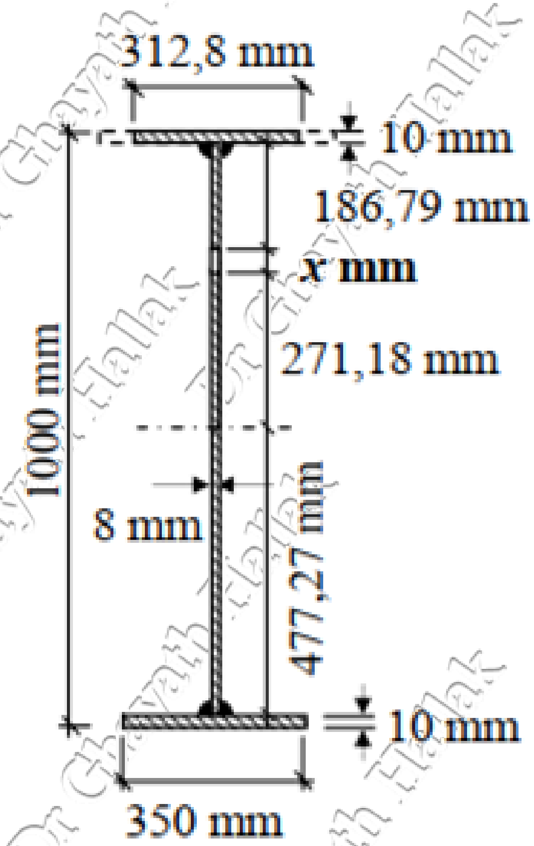
(ii) Consider bending

Cross-sectional area

A_{eff} = [3128 + 3500 + (980 × 8) - (44,76 × 8)]= 14109,92 mm^2

Location of centroid

z_{eff} = [(3128 × 995) + (186,79 × 8 × 896,61) + (748,45 × 8 × 384,23) + (3500 × 5)]/14109,92 = 479,83 mm



Second moment of area about the y-y axis:

I_{yy}= (312.8x10^3)/12+312.8x10x(1000-5-479.83)^2 + 186.79^{3}x8/12+186.79x8x(186.79/2+10-1000 + 479.83)^2 + (748.45^{3}x8)/12+ 748.45x8x (748.45/2+10-479.83)^2+350x10^3/12 + 350x10x (479.83-5)^2 = 2217.12 × 10^6 mm^4

z to the mid-point of the top flange = (995,0 - 479,83) = 515,17 mm

z to the mid-point of the bottom flange = (479,83 - 5,0) = 474,83 mm

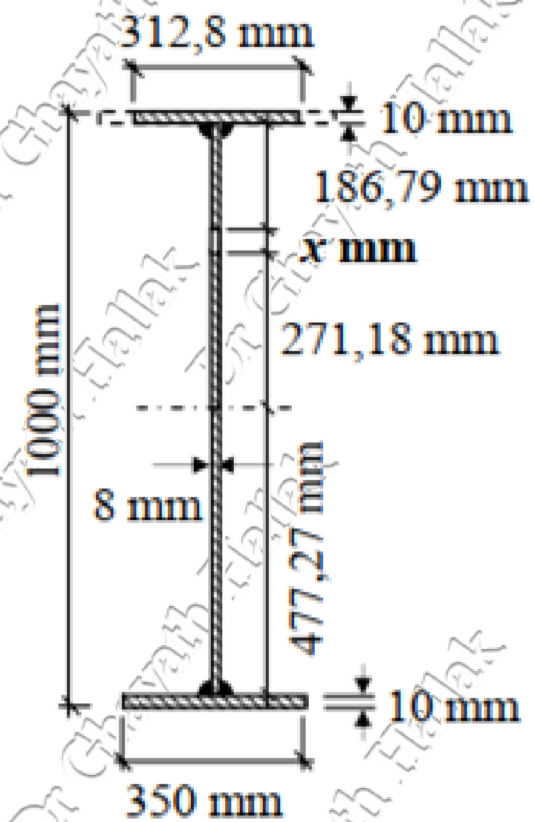
$W_{eff,y,top\ flange}$ = (2217,12 × 106)/515,17 = 4,30 × 10^6 mm^3

$W_{eff,y,bottom\ flange}$ = (2217,12 × 106)/474,83 = 4,67 × 10^6 mm^3

Effective areas – Class 4 -EN 1993-1-5: 2006 § 4.3

Example : Effective Cross-section Properties

An improved second approximation can be made by repeating the calculations using a stress ratio ψ based on the section properties from Figure shown



Summary: Effective section properties

Cross-sectional area (based on compression, see EN 1993-1-5: Clause 4.3(3)):

A_{eff} = 9224,2 mm^2

Shift of the centroid (based on compression, see EN 1993-1-1: Clause 6.2.9.3.(2)):

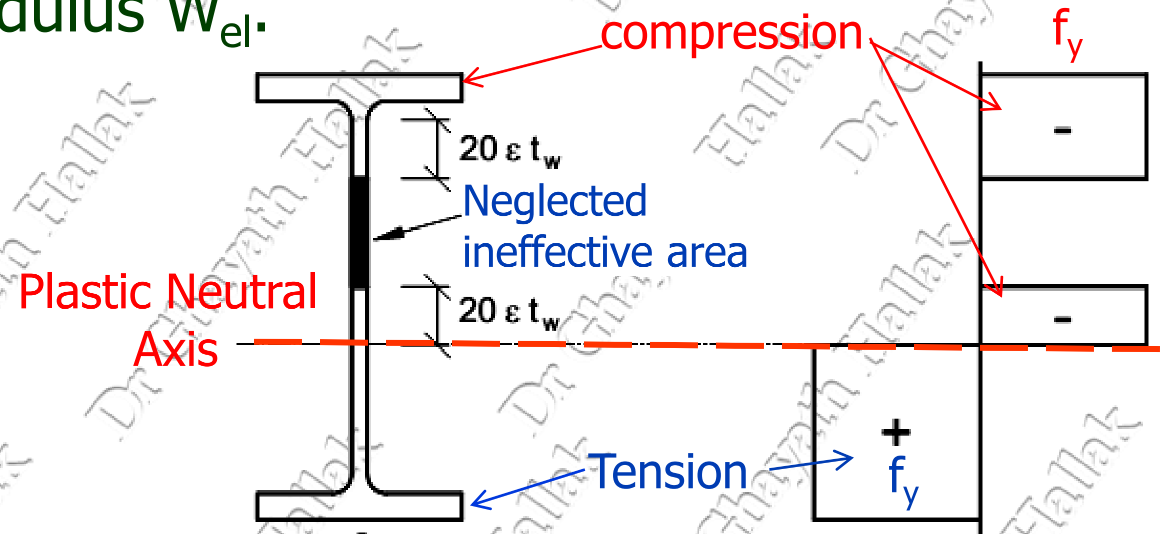
e_{Ny} = 22,85 mm

Minimum elastic section modulus (based on bending, see EN 1993-1-5: Clause 4.3(4)):

$W_{eff,y,min}$ = 4,30 × 10^6 mm^3

Effective properties of cross-sections with Class 3 webs and Class 1 or 2 flanges -EN 1993-1-1: 2005 § 6.2.2.4

Generally, a Class 3 cross-section (where the most slender element is Class 3) would assume an elastic distribution of stresses, and its bending resistance would be calculated using the elastic modulus W_{el}.

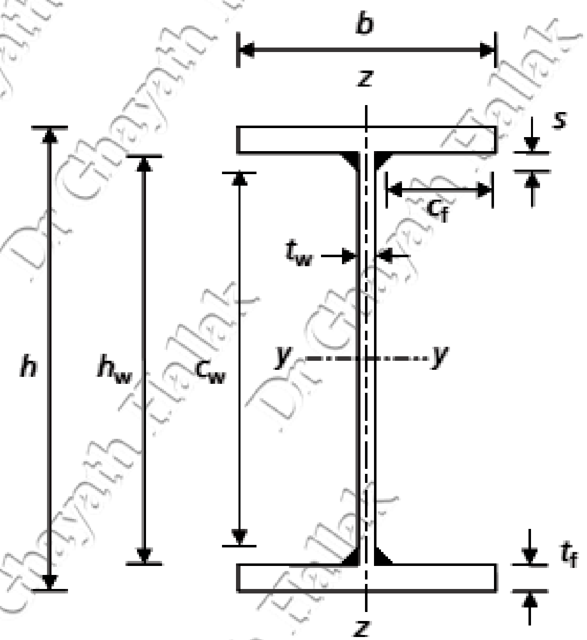


However, EN 1993-1-1: 2005 § 6.2.2.4 makes special allowances for cross-sections with Class 3 webs and Class 1 or 2 flanges by permitting the cross-sections to be classified as effective Class 2 cross-sections.

Effective properties of cross-sections with Class 3 webs and Class 1 or 2 flanges -EN 1993-1-1: 2005 § 6.2.2.4

Example : cross-section resistance in bending

A welded I section is to be designed in bending. The proportions of the section have been selected such that it may be classified as an effective Class 2 cross-section. The chosen section is of grade S275 steel, and has two 200 x16 mm flanges, an overall section height of 600 mm and a 6 mm web. The weld size (leg length) s is 6.0 mm. Assuming full lateral restraint, calculate the bending moment resistance.



b = 200.0 mm

t_f = 16.0 mm

h = 600.0 mm

t_w = 6.0 mm

s = 6.0 mm

$W_{el,\,y}$ = 2 124 838 mm^3

Effective properties of cross-sections with Class 3 webs and Class 1 or 2 flanges -EN 1993-1-1: 2005 § 6.2.2.4

Section properties

Since t_{max}=16mm Then f_y=275N/mm²

E = 210 000 N/mm²

Cross-section classification

$$\varepsilon = \sqrt{\frac{235}{f_y}} = \sqrt{\frac{235}{275}} = 0.92$$

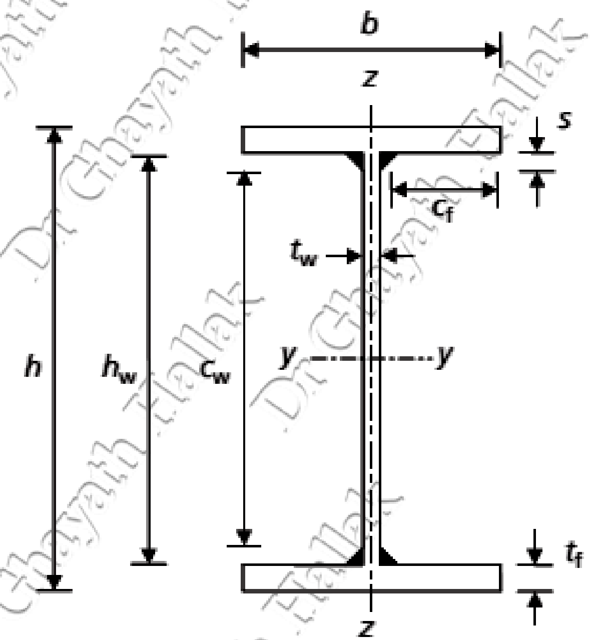
Outstand flanges (Table 5.2, sheet 2):

C_f= (b – t_w – 2s)/2 = 91.0 mm

C_f/t_f = 91/16= 5.69

Limit for Class 1 flange = 9ε= 8.32

8.32 > 6.86 ; flange is Class 1



b = 200.0 mm

t_f= 16.0 mm

h = 600.0 mm

t_w= 6.0 mm

s = 6.0 mm

$W_{el,y}$ = 2 124 838 mm³

Effective properties of cross-sections with Class 3 webs and Class 1 or 2 flanges -EN 1993-1-1: 2005 § 6.2.2.4

Cross-section classification

Web – internal part in bending (Table 5.2, sheet 1):

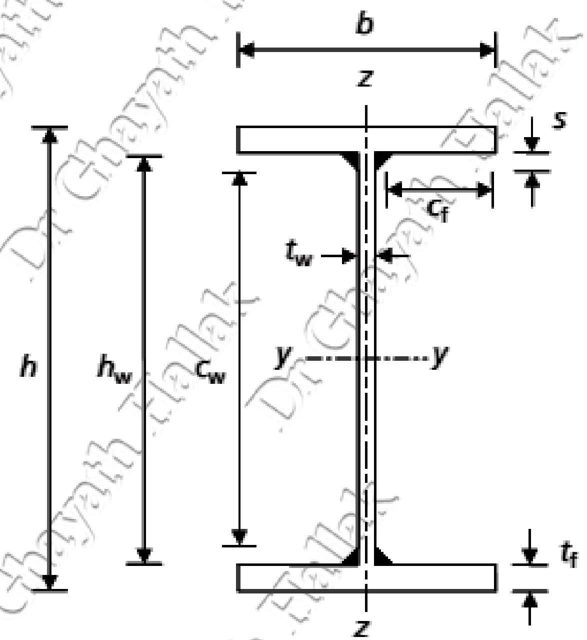
$c_w = h - 2t_f - 2s = 556.0\text{mm}$

$c_w/t_w = 556/6 = 92.7$

Limit for Class 2 web = $83\varepsilon = 76.4$

Limit for Class 3 web = $124\varepsilon = 114.6$

114.6> 92.7>76.4; web is Class 3



Overall cross-section classification is therefore Class 3.

b = 200.0 mm

t_f= 16.0 mm

h = 600.0 mm

t_w= 6.0 mm

s = 6.0 mm

$W_{el,\,y}$ = 2 124 838 mm^3

Effective properties of cross-sections with Class 3 webs and Class 1 or 2 flanges -EN 1993-1-1: 2005 § 6.2.2.4

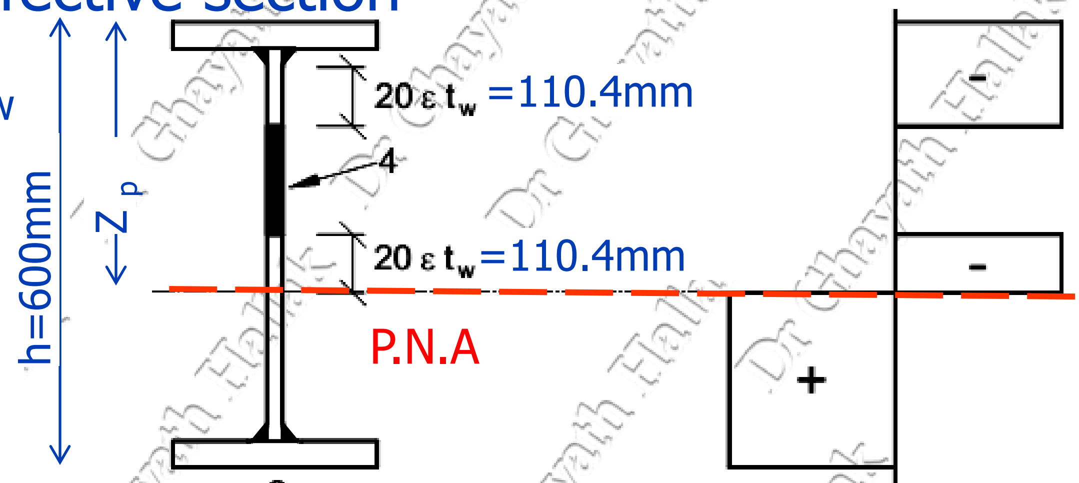
Plastic neutral axis of effective section

$b x t_f + 2x20xt_w x \varepsilon x t_w + s x t_w$

$= b x t_f + (h - t_f - z_p) x t_w$

$110.4x2x6 + 6x6 = (600 - 16 - z_p)x6$

$z_p = 357.2mm$



Plastic modulus of effective section

$$W_{pl,y,eff} = b\ t_f\ (z_p - 0.5t_f) + b\ t_f\ (h - z_p - 0.5t_f) + t_w\ s\ (z_p - t_f - 0.5s)\ + t_w\ (h - z_p - t_f)(h - z_p - t_f)/2 + (20\varepsilon t_w)\ t_w\ (20\varepsilon t_w)/2 + (20\varepsilon t_w)\ t_w\ [z_p - s - t_f - (20\varepsilon t_w)/2]$$

$$W_{pl,y,eff} = b\ t_f\ (h - t_f) + 0.5\ t_w\ (h - z_p - t_f)^2 + (20\varepsilon t_w)\ t_w\ [z_p - s - t_f] + t_w\ s\ (z_p - t_f - 0.5s) = 2\ 257\ 326\ mm^3$$

Effective properties of cross-sections with Class 3 webs and Class 1 or 2 flanges -EN 1993-1-1: 2005 § 6.2.2.4

Plastic Bending resistance of cross-section

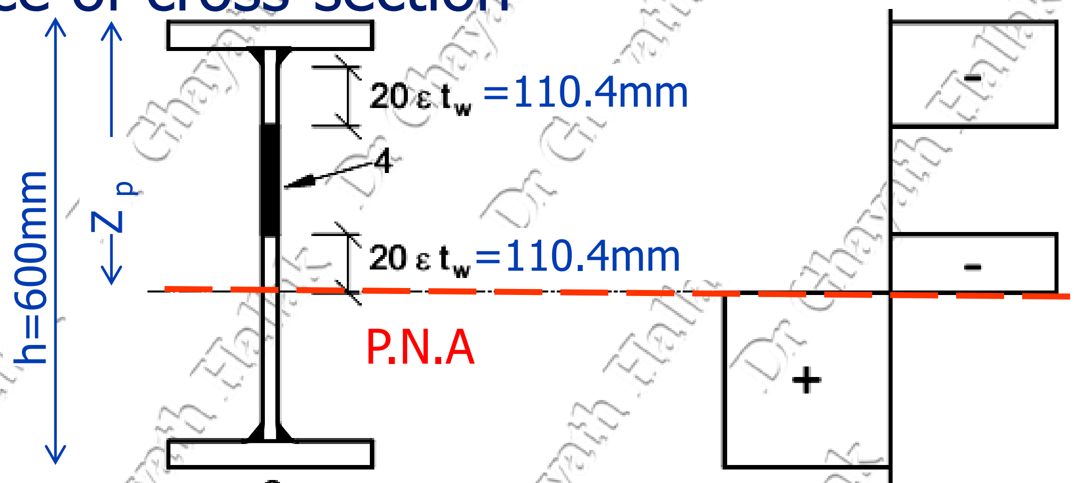
$M_{c,y,Rd} = M_{pl,y,Rd}$

$=W_{pl,y,eff}\ f_y/\gamma_{M0}$

for effective class 2 sections

$M_{c,y,Rd} = 2257326 \times 275/1$

$= \underline{620.76\ kN.m}$



Elastic section modulus

$W_{el,y} = [\ b\ h^3/12-(b-t_w)\ (h-2t_f)^3/12]/(0.5h) = 2\ 124\ 838 mm^3$

Elastic Bending resistance of cross-section

$M_{c,y,Rd} = M_{el,y,Rd} = W_{el,y}\ f_y/\gamma_{M0} = 2124838 \times 275/1 = \underline{584.3kN.m}$

Therefore, for the chosen section, use of the effective Class 2 plastic properties results in an increase in bending moment resistance of approximately 6%.