INTERNAL FORCES
Beams, Frames, Arches

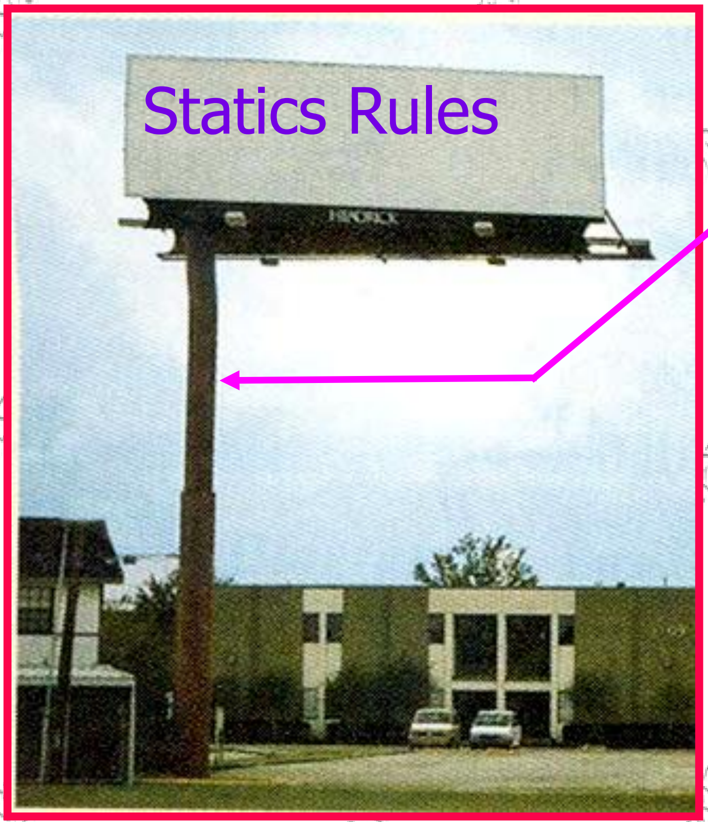
APPLICATIONS

These beams are used to support the roof of this gas station.

Why are the beams tapered? Is it because of the internal forces?

If so, what are these forces and how do we determine them?

APPLICATIONS



A fixed column supports this rectangular billboard.

Usually such columns are wider at the bottom than at the top. Why?

Is it because of the internal forces?

If so, what are they and how do we determine them?

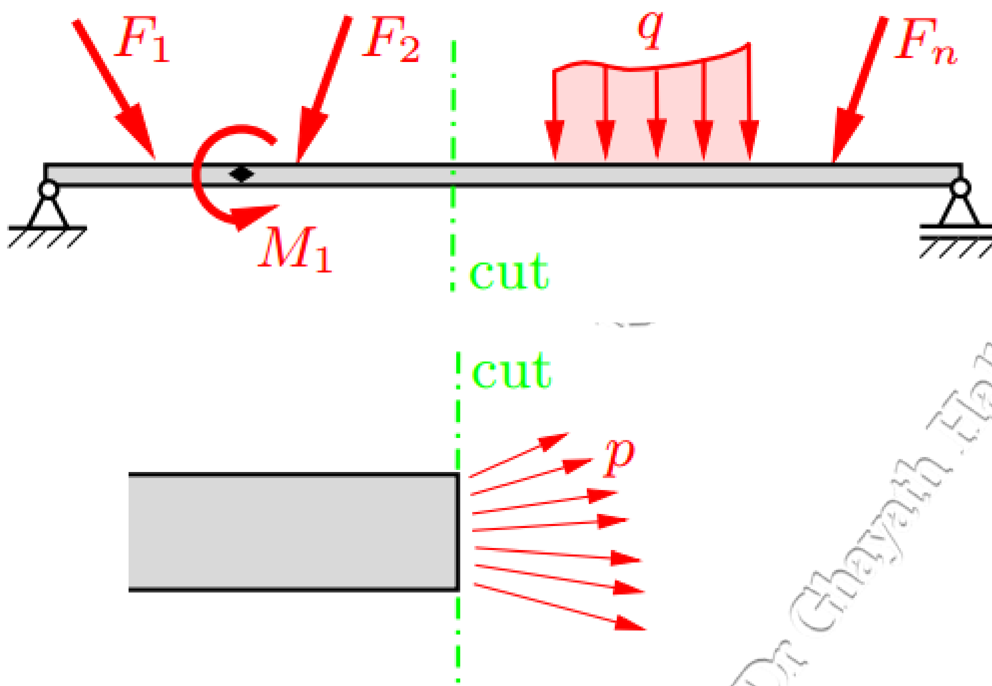
APPLICATIONS

The concrete supporting a bridge has fractured.

What might have caused the concrete to do this?

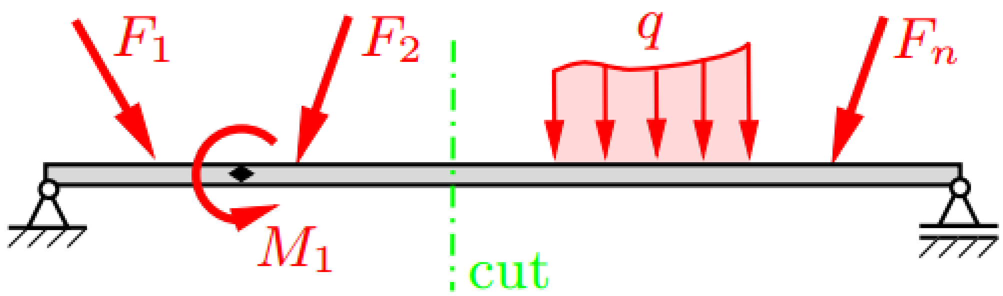
How can we analyze or design these structures to make them safer?

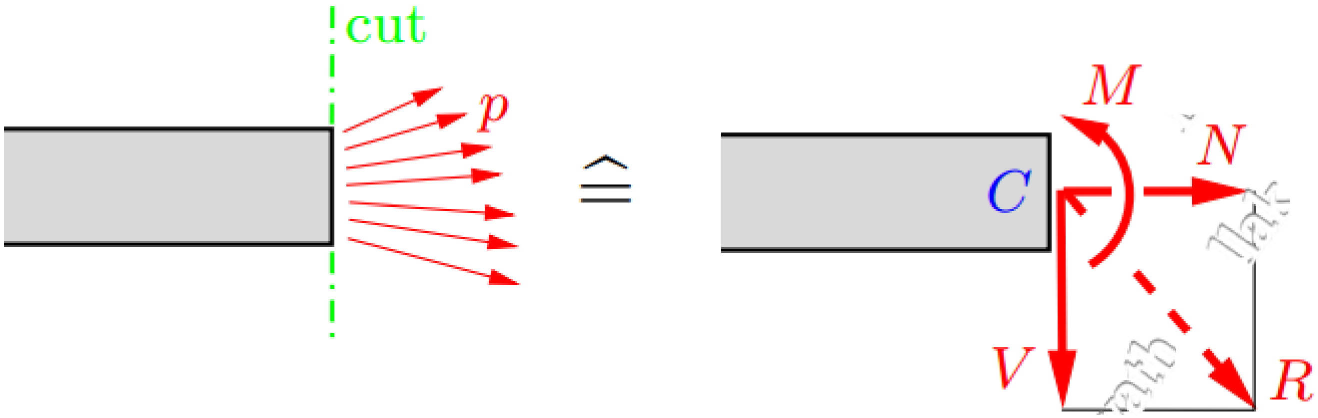
INTERNAL FORCES IN BEAMS



The internal forces in a beam can be made visible and thus accessible to calculation with the aid of a free-body diagram. Accordingly, we pass an imaginary section perpendicularly to the axis of the beam.

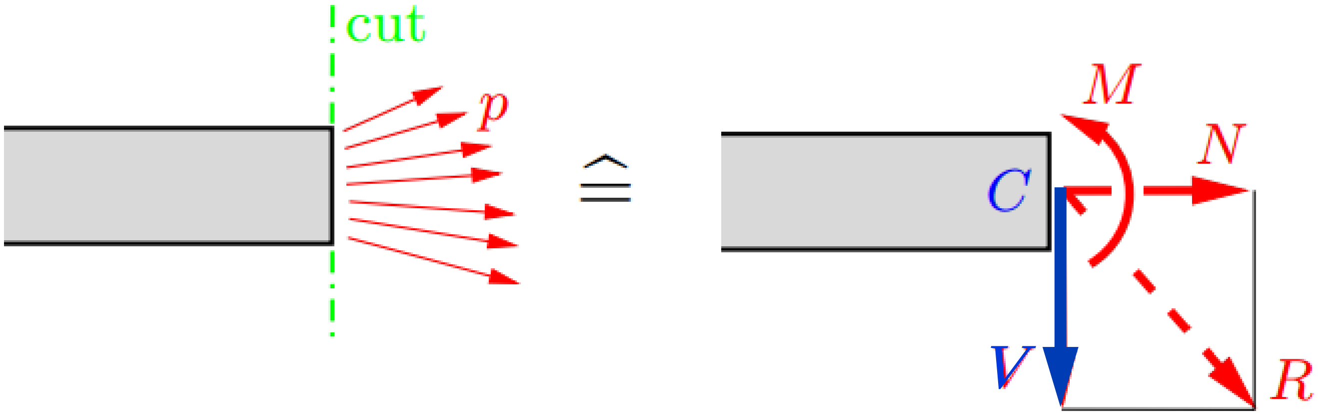
INTERNAL FORCES IN BEAMS





The internal forces p (forces per unit area) acting at the cross-section are distributed across the cross-sectional area. Their intensity is called stress.
it was shown previously that any force system can be replaced by a resultant force R acting at an arbitrary point C

INTERNAL FORCES IN BEAMS

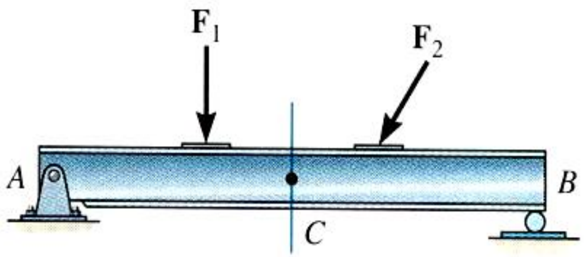


and a corresponding couple $M_{(C)}$. When carrying this out, we choose the centroid C of the cross-sectional area as the reference point of the reduction.

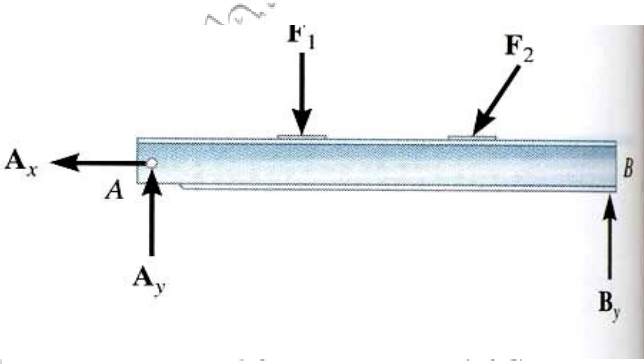
The resultant force R is resolved into its components N (normal to the cross-section, in the direction of the axis of the beam) and V (in the cross section, orthogonal to the axis of the beam). The quantities N, V and M are called the stress resultants. In particular,

N is called the *normal force*, V is the *shear force* and M is the *bending moment*

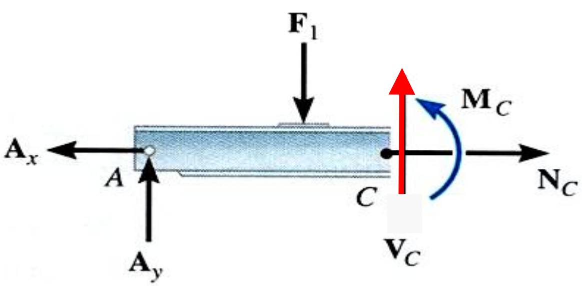
INTERNAL FORCES IN BEAMS



The design of any structural member requires finding the forces acting within the member to make sure the material can resist those loads.

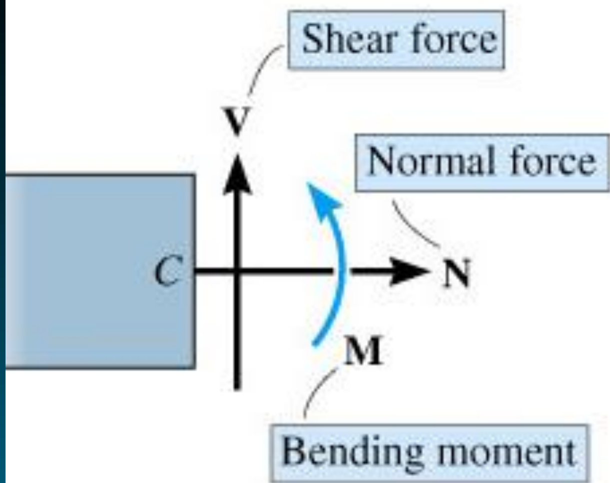


For example, we want to determine the internal forces acting on the cross section at C. First, we first need to determine the support reactions.

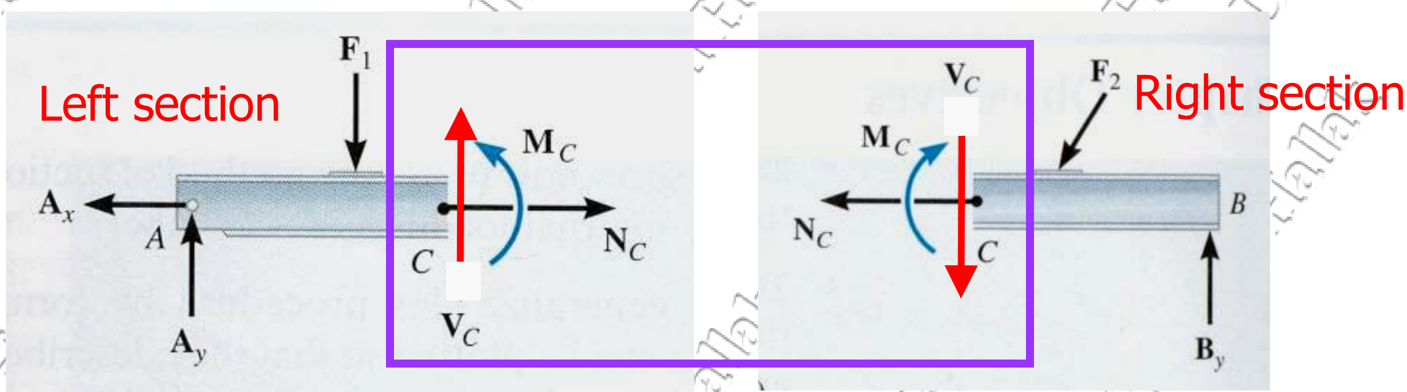


Then we need to cut the beam at C and draw a FBD of one of the halves of the beam. This FBD will include <u>the internal forces acting at C.</u> Finally, we need to solve for these unknowns using the E-of-E.

NAL FORCES IN BEAMS

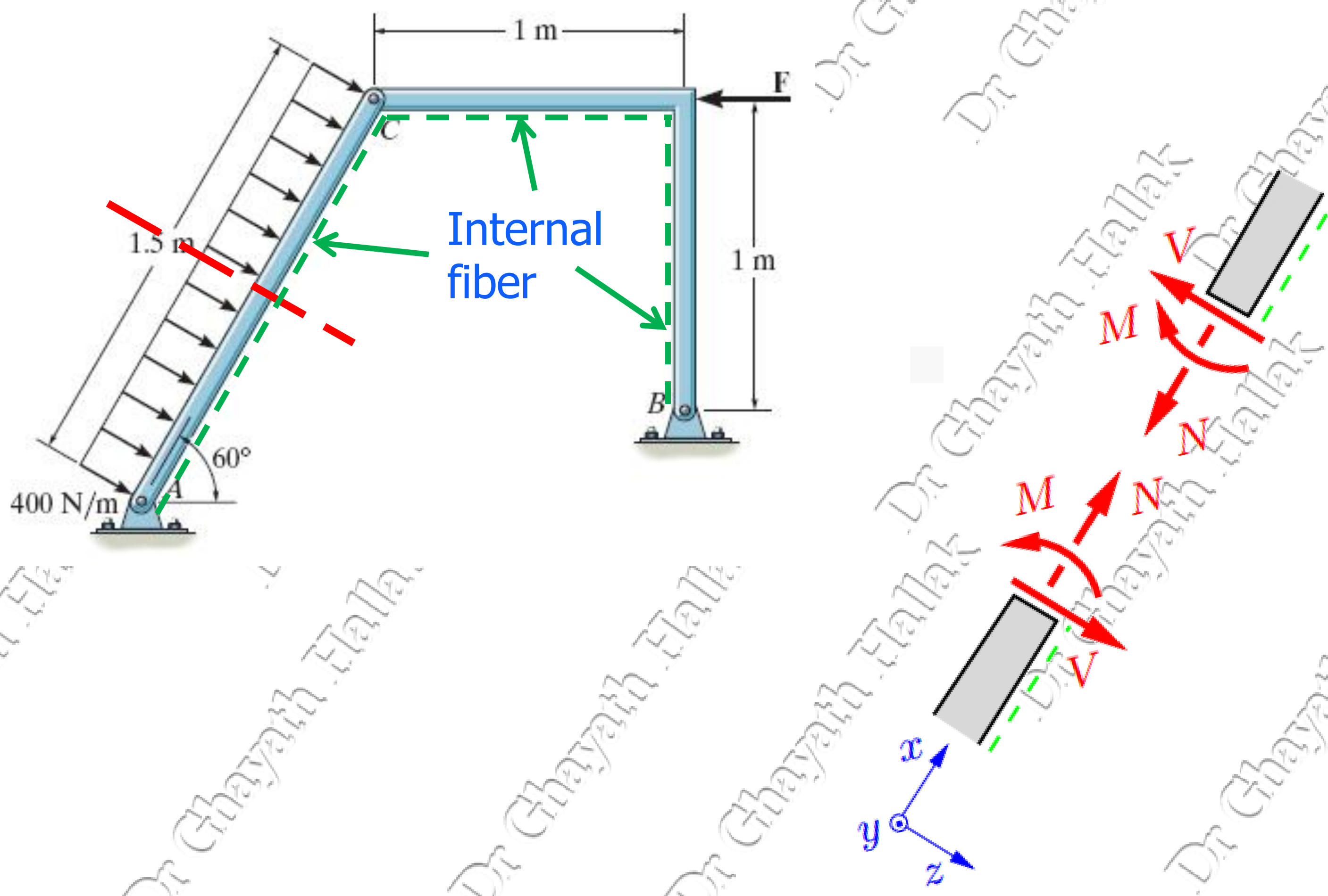


In two-dimensional cases, typical internal loads are normal or axial forces (N, acting perpendicular to the section), shear forces (V, acting along the surface), and the bending moment (M).



The loads on the left and right sides of the section at C are equal in magnitude but opposite in direction. This is because when the two sides are reconnected, the net loads are zero at the section.

INTERNAL FORCES IN FRAMES



STEPS FOR DETERMINING INTERNAL FORCES

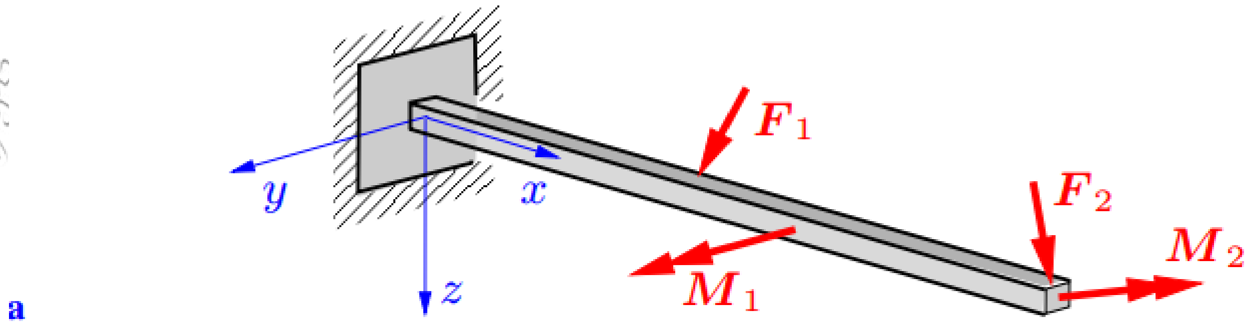
1. Take an imaginary cut at the place where you need to determine the internal forces. Then, decide which resulting section or piece will be easier to analyze.

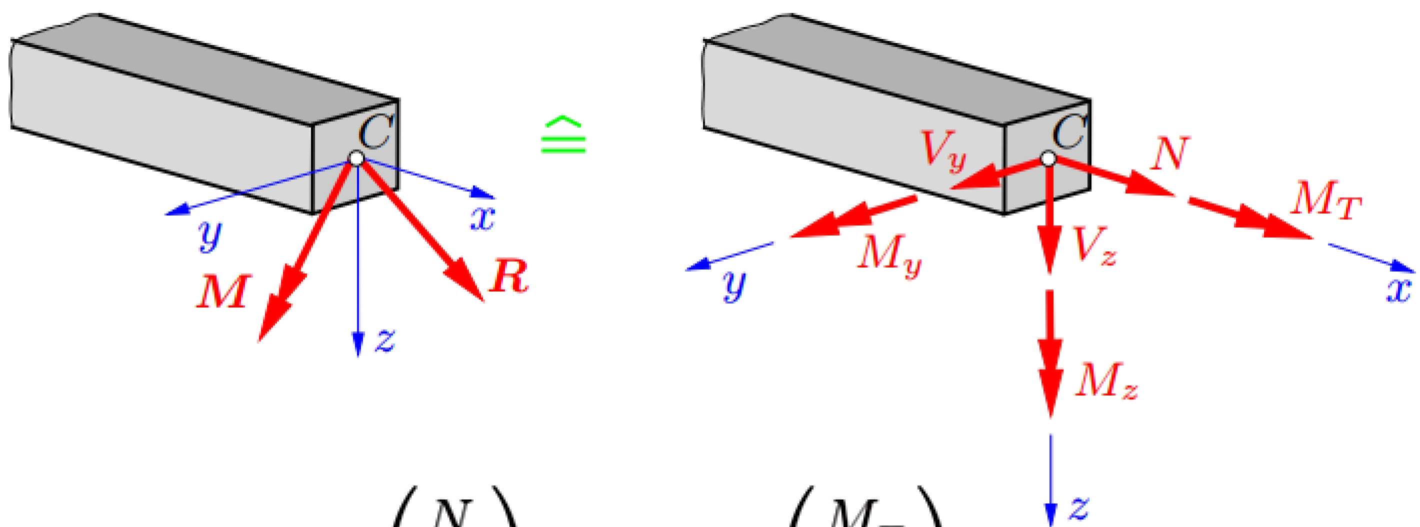
2. If necessary, determine any support reactions or joint forces you need by drawing a FBD of the entire structure and solving for the unknown reactions.

3. Draw a FBD of the piece of the structure you've decided to analyze. Remember to show the N, V, and M loads at the "cut" surface.

4. Apply the E-of-E to the FBD (drawn in step 3) and solve for the unknown internal loads.

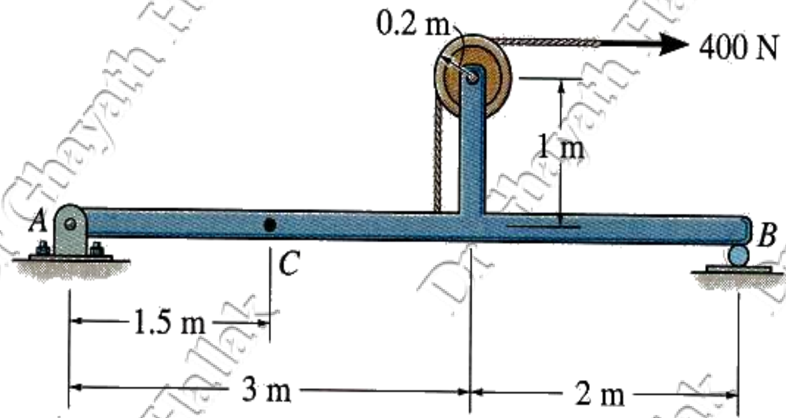
INTERNAL FORCES IN SPATIAL STRUCTURES





$$\boldsymbol{R} = \begin{pmatrix} N \\ V_y \\ V_z \end{pmatrix}, \quad \boldsymbol{M} = \begin{pmatrix} M_T \\ M_y \\ M_z \end{pmatrix}$$

EXAMPLE



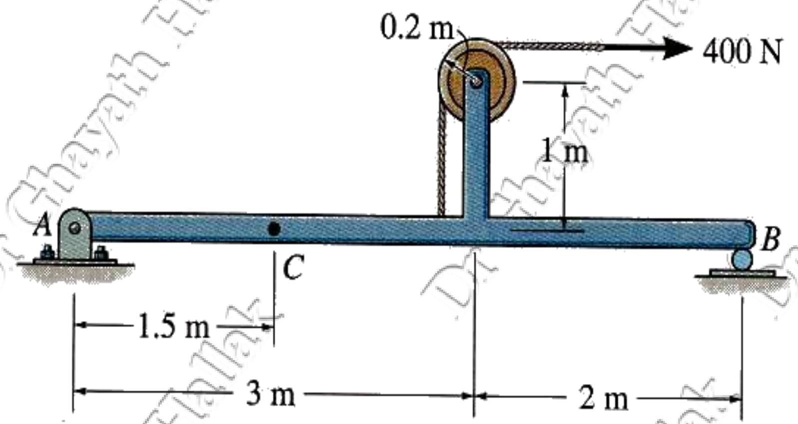
Given: The loading on the beam.

Find: The internal forces at point C.

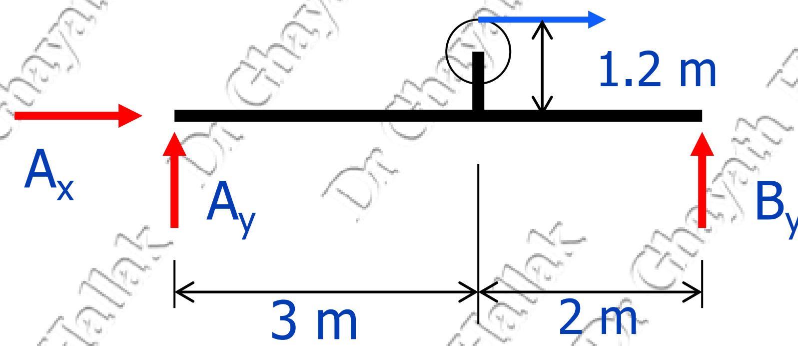
Solution

1. Plan on taking the imaginary cut at C. It will be easier to work with the left section (point A to the cut at C) since the geometry is simpler.
2. We need to determine A_x and A_y using a FBD of the entire frame.

EXAMPLE



FBD:



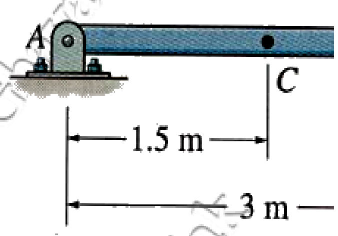
Applying the E-of-E to this FBD, we get

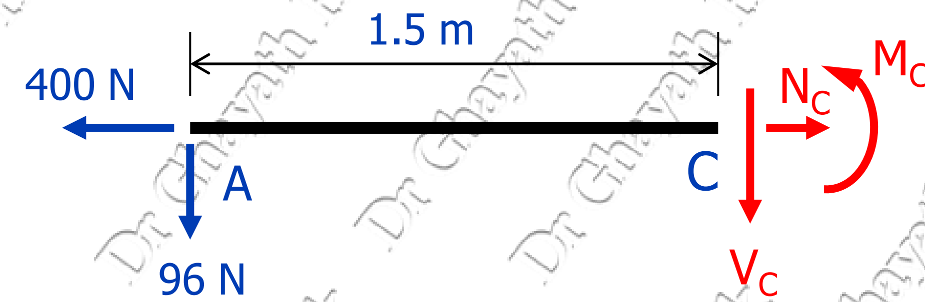
$\rightarrow + \Sigma F_x \quad = \quad A_x + 400 = 0 \;; \qquad A_x = -400 \text{ N}$

$\circlearrowleft + \Sigma M_B = \quad -A_y(5) - 400\,(1.2) = 0 \;; \qquad A_y = -96 \text{ N}$

3. Now draw a FBD of the left section. Assume directions for V_C, N_C and M_C.

EXAMPLE





4. Applying the E of E to this FBD, we get

$\rightarrow + \Sigma F_x = N_C - 400 = 0;$ $N_C = 400$ N

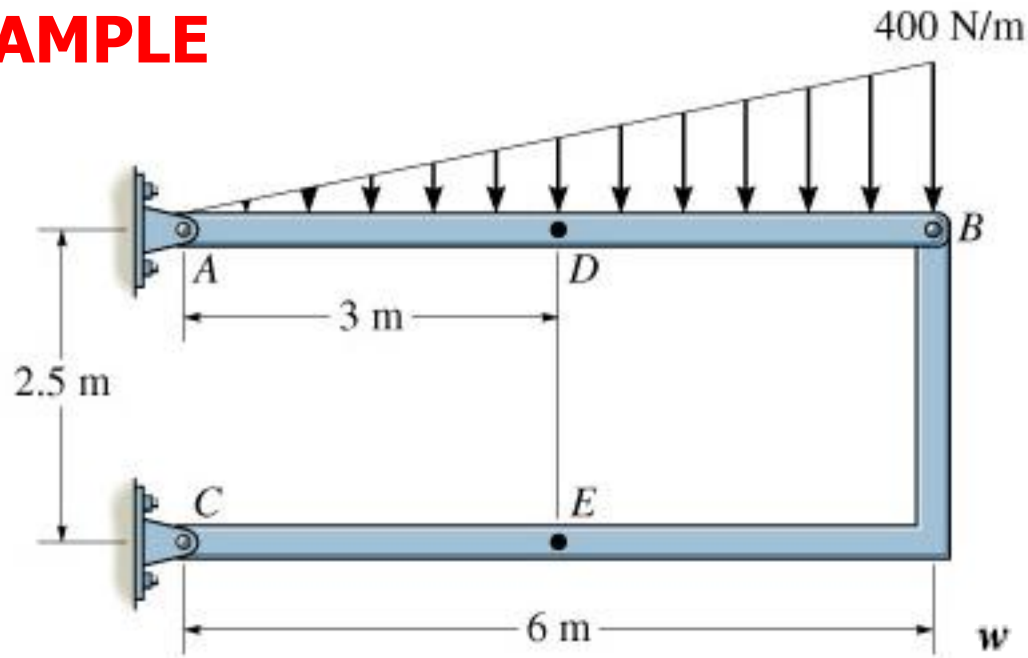
$\uparrow + \Sigma F_y = - V_C - 96 = 0;$ $V_C = - 96$N

$\circlearrowleft + \Sigma M_C = 96\,(1.5) + M_C = 0\,;$ $M_C = -144$ N m

EXAMPLE

Determine the normal force, shear force, and moment at a section passing through point *D* of the two-member frame.

w = 400N/m, a = 2.5 m, b = 3 m, c = 6 m



Solution:

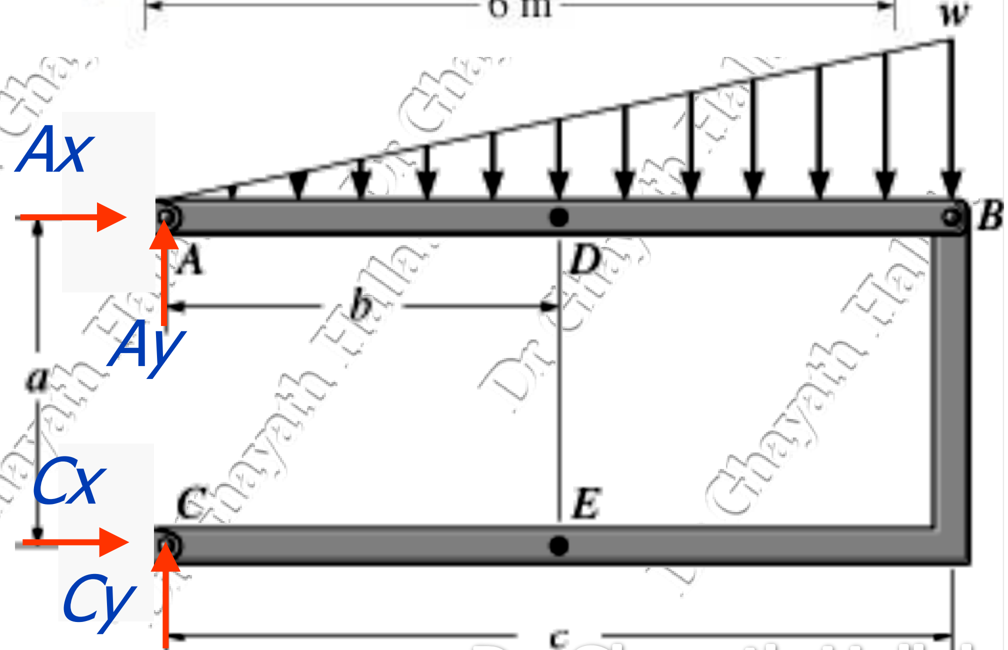
$\circlearrowleft \Sigma M_C = 0;$

$Ax \times 2.5 + (400\times 6/2) \times (2\times 6/3) = 0$

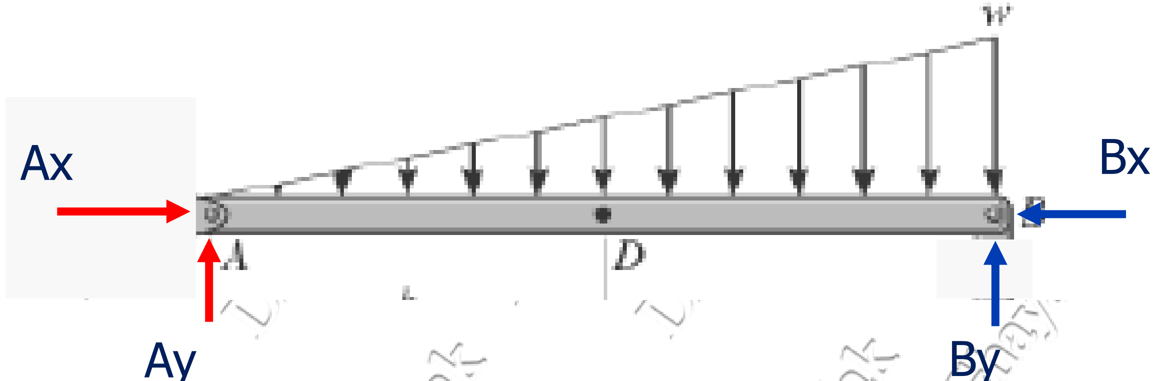
$Ax = -\,1920 \text{ N}$

$\rightarrow + \Sigma F_x = 0;\ Cx + Ax = 0$

$Cx = -\,Ax = 1920 \text{ N}$



EXAMPLE



$\curvearrowright$ $\Sigma M_A = 0;$ ⇒ $By \times 6 - (400 \times 6/2) \times (2 \times 6/3) = 0$

$By = 800\ N$

$\uparrow + \Sigma F_y = 0;$ ⇒ $Ay + 800 - (400 \times 6/2) = 0$

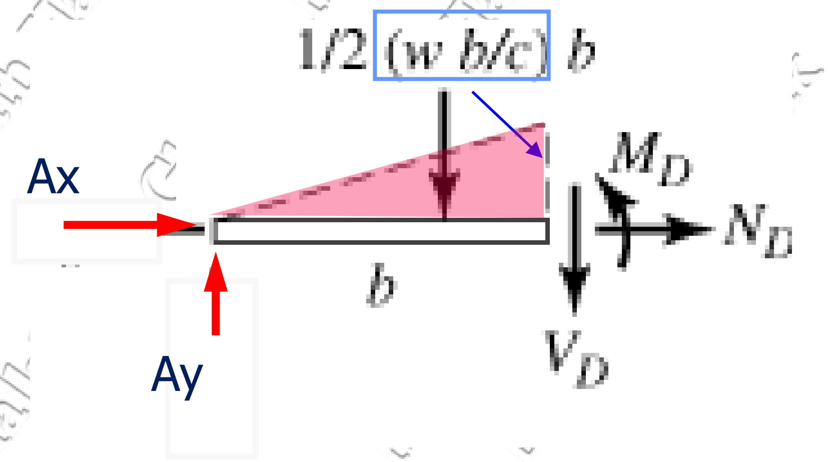
$Ay = 400\ N$

$\rightarrow + \Sigma F_x = 0;$ ⇒ $Ax - Bx = 0$

$Bx = Ax = -1920\ N$

EXAMPLE

Internal Forces at point D:



$\xrightarrow{+} \Sigma F_x = 0;\ N_D + A_x = 0$

$N_D = -A_x \Rightarrow N_D = 1920$ N

$\uparrow + \Sigma F_y = 0;$

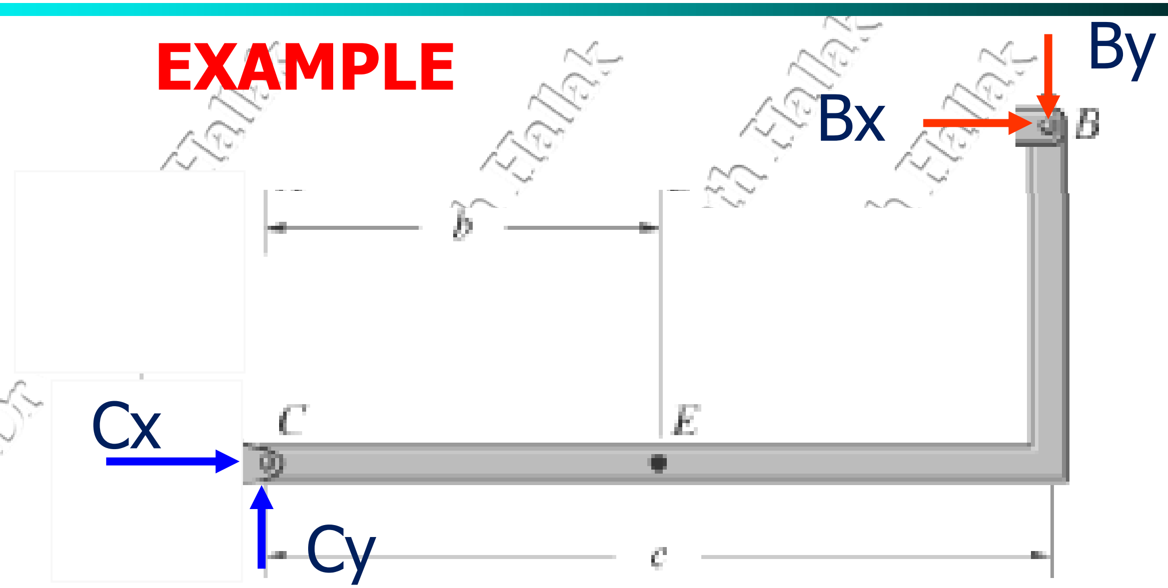
$A_y - V_D - 1/2 \times (400 \times 3/6) \times 3 = 0$

$\Rightarrow V_D = 100$ N

$\circlearrowright \Sigma M_D = 0;\quad -A_y \times 3 + (400 \times 3/6) \times (3/2 \times 1/3 \times 3) + M_D = 0$

$\Rightarrow M_D = 900$ N.m

EXAMPLE



$\rightarrow + \Sigma Fx = 0; \quad Cx + Bx = 0 \Rightarrow Cx = -Bx = 1920$ N

$\uparrow + \Sigma Fy = 0; \quad Cy - By = 0 \Rightarrow Cy = By = 800$ N

$\rightarrow + \Sigma Fx = 0; \quad Cx + N_E = 0 \Rightarrow$

$N_E = -Cx = -1920$ N

$\uparrow + \Sigma Fy = 0; \quad Cy - VE = 0 \Rightarrow$

$VE = Cy = 800$ N

$\curvearrowright \Sigma M_E = 0; \quad -Cy \times 3 + M_E = 0 \qquad M_E = 2400$ N.m