Faculty of Civil Engineering CALCULATION SHEET	Job No.	Sheet 1 of 8	Rev:
	Job title		
	Subject: Example- Restrained Beam		
	Client: 4Th EAR- Faculty of civil Engineering	Made by: SCI	Date:
		Checked by: Dr. G. Hallak	Date:

The beam shown in Figure 2.1 is fully laterally restrained along its length and has bearing lengths of 50 mm at the unstiffened supports and 75 mm under the point load. Design the beam in S275 steel for the loading shown below.

References are to BS EN 1993-1-1: 2005, including its National Annex, unless otherwise stated

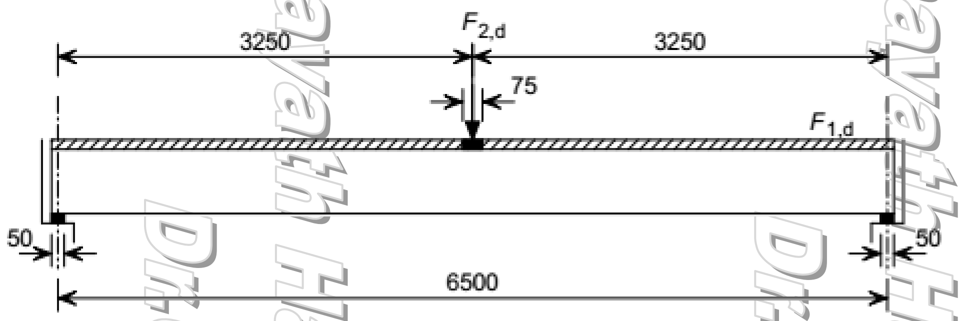


Figure 2.1

2.2 Actions (loading)

2.2.1 Permanent actions

Uniformly distributed load (including self weight) g_1 = 15 kN/m

Concentrated load G_2 = 40 kN

2.2.2 Variable actions

Uniformly distributed load q_1= 30 kN/m

Concentrated load Q_2= 50 kN

The variable actions are not due to storage and are not independent of each other.

2.2.3 Partial factors for actions

BS EN 1990 A1.3.1(4)

Table NA.A1.2(B)

Partial factor for permanent actions $\gamma_G = 1.35$

Partial factor for variable actions $\gamma_Q = 1.50$

Reduction factor $\xi = 0.925$

2.2.4 Design values of combined actions for Ultimate Limit State

Use Expression (6.10) or the less favourable combination from Expression (6.10a) and (6.10b). The UK National Annex to BS EN 1990 allows the designer to choose which of those options to use.

BS EN 1990 Table NA.A1.2(B)

$$\gamma_{Gj,sup} G_{j,sup} + \gamma_{Gj,inf} G_{j,inf} + \gamma_{Q,1}\psi_{0,1} Q_1 + \gamma_{Q,i}\psi_{0,i} Q_i \qquad (6.10a)$$

$$\xi\gamma_{Gj,sup} G_{j,sup} + \gamma_{Gj,inf} G_{j,inf} + \gamma_{Q,1} Q_1 + \gamma_{Q,i}\psi_{0,i} Q_i \qquad (6.10b)$$

UDL (including self weight)

$F1,d = \xi\gamma_G\, g_1 + \gamma_Q\, q_1 = (0.925 \times 1.35 \times 15) + (1.5 \times 30) = 63.7$ kN/m

Concentrated load

$F2,d = \xi\gamma_G G_2 + \gamma_Q Q_2 = (0.925 \times 1.35 \times 40) + (1.5 \times 50) = 125.0$ kN

2.3 Design bending moments and shear forces

Span of beam L = 6500 mm

Maximum design bending moment occurs at mid-span

$$M_{Ed} = \frac{F_{1,d}L^2}{8} + \frac{F_{2,d}L}{4} = \frac{63.7\times6.5^2}{8} + \frac{125\times6.5}{4} = 539.5 \text{ kNm}$$

Maximum design shear force occurs at the supports

$$V_{Ed} = \frac{F_{1,d}L}{2} + \frac{F_{2,d}}{2} = \frac{63.7\times6.5}{2} + \frac{125}{2} = 269.5 \text{ kN}$$

Design shear force at mid-span

$$V_{c,Ed} = V_{Ed} - \frac{F_{1,d}L}{2} = 269.50 - \frac{63.7\times6.5}{2} = 62.5 \text{ kN}$$

2.4 Section properties

Trial section can be calculated as follows:

$M_{c,Rd}=W_{pl,y}\, f_y/\gamma_{M0}=M_{Ed} \Rightarrow W_{pl,y}= M_{Ed}/(f_y/\gamma_{M0}) \Rightarrow W_{pl,y}= 539.5\text{x}10^6/275=1962\text{cm3}$

Chose from the UKB section tables a section has Wpl,y >1962cm3

Try section 533 × 210 × 92 UKB in S275 with $W_{pl,y}$ = 2 360 cm^3

From section property tables:

Depth	h = 533.1 mm
Width	b = 209.3 mm
Web thickness	t_w = 10.1 mm
Flange thickness	t_f = 15.6 mm
Root radius	r = 12.7 mm
Depth between flange fillets	d = 476.5 mm
Second moment of area, y-y axis	I_y = 55 200 cm^4
Plastic modulus, y-y axis	$W_{pl,y}$ = 2 360 cm^3
Area	A = 117 cm^2
Modulus of elasticity	E = 210 000 N/mm^2

For S275 steel and $t \leq 16$ mm

Yield strength $f_y = R_{eH}$ = 275 N/mm^2 — BS EN 10025-2 Table 7

2.5 Cross section classification

$$\varepsilon = \sqrt{\frac{235}{f_y}} = \sqrt{\frac{235}{275}} = 0.92$$

Table 5.2

Outstand of compression flange

$$c = \frac{b - t_w - 2r}{2} = \frac{209.3 - 10.1 - (2 \times 12.7)}{2} = 86.90 \text{ mm}$$

$$\frac{c}{t_f} = \frac{86.90}{15.6} = 5.57$$

The limiting value for Class 1 is $\frac{c}{t_f} \leq 9\varepsilon = 9 \times 0.92 = 8.28$

5.57 < 8.28

Therefore the flange is Class 1 under compression.

Web subject to bending

$c = d = 476.5$ mm

$$\frac{C}{t_w} = \frac{476.5}{10.1} = 47.18$$

The limiting value for Class 1 is $\frac{c}{t_w} \leq 72\varepsilon = 72 \times 0.92 = 66.24$

47.18 < 66.24

Therefore the web is Class 1 under bending.

Therefore the section is Class 1 under bending.

2.6 Partial factors for resistance

$\gamma_{M0} = 1.0$

$\gamma_{M1} = 1.0$ NA.2.15

2.7 Cross-sectional resistance

2.7.1 Shear buckling

The shear buckling resistance for webs should be verified according to Section 5 of BS EN 1993-1-5 if: 6.2.6(6)

$$\frac{h_w}{t_w} > \frac{72\varepsilon}{\eta}$$

$\eta = 1.0$ BS EN 1993-1-5 NA.2.4

$h_w = h - 2t_f = 533.1 - (2 \times 15.6) = 501.9$ mm

$$\frac{h_w}{t_w} = \frac{501.9}{10.1} = 49.7$$

$$\frac{72\varepsilon}{\eta} = \frac{72 \times 0.92}{1.0} = 66.2$$

49.7 < 66.2

Therefore the shear buckling resistance of the web does not need to be verified.

2.7.2 Shear resistance

Verify that: (6.2.6(1))

$$\frac{V_{Ed}}{V_{c,Rd}} \leq 1.0 \quad \text{Eq (6.17)}$$

$$V_{c,Rd} = V_{pl,Rd} = \frac{A_V \times f_y}{\gamma_{M0} \times \sqrt{3}}$$

$A_V = A - 2 t_f b + (t_w + 2r) t_f \geq \eta h_w t_w$ (6.2.6(3))

$A_V = 117\times10^2 - (2\times209.3\times15.6) + 15.6\times[10.1 + (2\times12.7)] = 5723.6 mm^2$

$\eta h_w t_w = 1.0\times501.9\times10.1 = 5069.2 mm^2$

Therefore,

$Av = 5723.6 mm^2$

$$V_{c,Rd} = V_{pl,Rd} = \frac{5723.6 \times 275}{1.0 \times \sqrt{3}} = 908.7 kN$$

$$\frac{V_{Ed}}{V_{c,Rd}} = \frac{269.5}{908.7} = 0.3 \leq 1.0$$

Therefore the shear resistance of the section is adequate.

2.7.3 Resistance to bending

Verify that: (6.2.5(1) Eq (6.12))

$$\frac{M_{Ed}}{M_{c,Rd}} \leq 1.0$$

At the point of maximum bending moment (mid-span), verify whether the shear force will reduce the bending resistance of the cross section. (6.2.8(2))

$$\frac{V_{c,Rd}}{2} = \frac{908.7}{2} = 454.4\ kN$$

Shear force at maximum bending moment $V_{c,Ed} = 62.5$ kN

62.5 kN < 454.5 kN

Therefore **no reduction** in bending resistance due to shear is required.(Low shear)

The design resistance for bending for Class 1 and 2 cross sections is: (6.2.5(2))

$$M_{c,Rd} = M_{pl,Rd} = \frac{W_{pl,y} \times f_y}{\gamma_{M0}} = \frac{2360 \times 10^3 \times 275}{1.0} \times 10^{-6} = 649 kN.m \quad \text{Eq (6.13)}$$

$$\frac{M_{Ed}}{M_{c,Rd}} = \frac{539.5}{649} = 0.83 < 1.0 \quad \text{6.2.5(1) Eq (6.12)}$$

Therefore the bending moment resistance is adequate.

2.7.4 Resistance of the web to transverse forces

This verification is only required when there is bearing on the beam. BS EN 1993-1-1 does not give design verifications for the resistance of webs, designers are referred to BS EN 1993-1-5.

References Given in Section 2.7.4 refer to BS EN 1993-1-5

Verify that:

$$\eta_2 = \frac{F_{Ed}}{F_{Rd}} \leq 1.0$$

where:

F_{Ed} is the design transverse force – here this is taken to be the design shear force at the supports as these have the smallest bearing lengths (50 mm)

$$F_{Rd} = \frac{f_{yw} \times L_{eff} \times t_w}{\gamma_{M1}} \; (Design\ Resistance)$$

L_{eff} is the effective length for resistance to transverse forces, given by,

$L_{eff} = \chi_F\, \ell_y$

$$\chi_F = \frac{0.5}{\bar{\lambda}_F} \leq 1.0$$ 6.4(1) Eq (6.3)

$$\bar{\lambda}_F = \sqrt{\frac{f_{yw} \times l_y \times t_w}{F_{cr}}}$$ 6.4(1) Eq (6.4)

Determine ℓ_y and $\bar{\lambda}_F$

The force is applied to one flange adjacent to an unstiffened end and the compression flange is restrained, therefore it is Type c. — 6.1(2)c) & Figure 6.1

The length of stiff bearing on the flange is the length over which the load is effectively distributed at a slope of 1:1. However, s_s should not be greater than h_w. — 6.3(1) & Figure 6.2

For a slope of 1:1 S_s = 50 mm < h_w = 501.9 mm

Therefore,

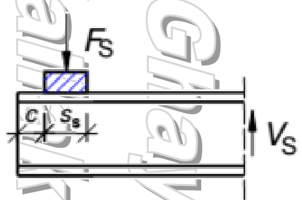
S_s = 50 mm

For webs without longitudinal stiffeners k_F should be obtained from Figure 6.1 For Type c — 6.4(2) Figure 6.1

$$k_F = 2 + 6\left(\frac{S_s + c}{h_w}\right) \leq 6$$

c = 0 mm

$$k_F = 2 + 6\left(\frac{50+0}{501.9}\right) = 2.6 < 6$$



For Type c) ℓ_y is the smallest of the values determined from Equations (6.10), (6.11) and (6.12).

$$l_y = S_s + 2t_f\left(1 + \sqrt{m_1 + m_2}\right) \text{ but } l_y \leq \text{distance between adjacent stiffeners}$$ 6.5(2) Eq (6.10)

As there are no stiffeners in the beam in this example neglect the above limit for ℓ_y.

Or

$$l_{y1} = l_e + t_f\sqrt{\frac{m_1}{2} + \left(\frac{l_e}{t_f}\right)^2 + m_2}$$ 6.5(3) Eq (6.11)

Or

$$l_{y2} = l_e + t_f\sqrt{m_1 + m_2}$$ Eq (6.12)

where:

$$l_e = \frac{k_f E t_w^2}{2 f_{yw}\ h_w} \leq S_s + c$$

$$l_e = \frac{2.6 \times 210000 \times 10.1^2}{2 \times 275 \times 501.9} = 201.77mm > S_s + c = 50.0mm$$ Eq (6.13)

Therefore

$$l_e = S_s + c = 50.0mm$$

Factors m_1 and m_2 are determined as follows:

$$m_1 = \frac{f_{yf}\ b_f}{f_{yw}\ t_w} = \frac{275 \times 209.3}{275 \times 10.1} = 20.72$$

$$m_2 = 0.02\left(\frac{h_w}{t_f}\right)^2 = 0.02\left(\frac{501.9}{15.6}\right)^2 = 20.70\ \ when\ \bar{\lambda}_F > 0.5$$

Or

$$m_2 = 0\ \ when\ \ \bar{\lambda}_F \leq 0.5$$

a) First, consider $m_2 = 0$

$$l_y = 50\ + 2 \times 15.6 \times \left(1 + \sqrt{20.72 + 0}\right)\ = 223.22mm$$

$$l_{y1} = 50 + 15.6\sqrt{\frac{20.72}{2} + \left(\frac{50}{15.6}\right)^2 + 0} = 120.86mm$$

$$l_{y2} = 50 + 15.6\sqrt{20.72 + 0} = 121.01mm$$

As 120.86 mm < 121.01 mm < 223.22 mm

ℓ_y = 120.86 mm

$$\bar{\lambda}_F = \sqrt{\frac{f_{yw} \times l_y \times t_w}{F_{cr}}}$$ 6.4(1) Eq (6.4)

$$F_{cr} = 0.9\frac{k_f E\ t_w^3}{h_w} = 0.9 \times 2.6 \times 210000 \times \frac{10.1^3}{501.9} \times 10^{-3} = 1008.7kN$$ 6.4(1) Eq (6.5)

Therefore

$$\bar{\lambda}_F = \sqrt{\frac{f_{yw} \times l_y \times t_w}{F_{cr}}} = \sqrt{\frac{275 \times 120.86 \times 10.1}{1008.7 \times 10^3}} = 0.58 > 0.5$$

As $\bar{\lambda}_F > 0.5$, m_2 must be determined and ℓ_y recalculated with m_2 = 20.70

b) Recalculate for m_2 = 20.70

$$l_y = 50\ + 2 \times 15.6 \times \left(1 + \sqrt{20.72 + 20.70}\right)\ = 282.0mm$$

$$l_{y1} = 50 + 15.6\sqrt{\frac{20.72}{2} + \left(\frac{50}{15.6}\right)^2 + 20.70} = 150.29mm$$

$l_{y2} = 50 + 15.6\sqrt{20.72 + 20.70} = 150.40mm$

As 150.29 mm < 150.40 mm < 282.00 mm

$\ell_y = 150.29$ mm

$$\bar{\lambda}_F = \sqrt{\frac{f_{yw} \times l_y \times t_w}{F_{cr}}} = \sqrt{\frac{275 \times 150.29 \times 10.1}{1008.7 \times 10^3}} = 0.64 > 0.5$$

As 0.64 > 0.5, $\bar{\lambda}_F = 0.64$

Determine χ_F

$$\chi_F = \frac{0.5}{\bar{\lambda}_F} = \frac{0.5}{0.64} = 0.78 < 1.0$$

Determine L_{eff}

$$L_{eff} = \chi_F\, \ell_y = 0.78 \times 150.29 = 117.23 \text{ mm}$$

Determine F_{Rd}

$$F_{Rd} = \frac{f_{yw} \times L_{eff} \times t_w}{\gamma_{M1}} = \frac{275 \times 117.23 \times 10.1}{1.0} \times 10^3 = 325.6kN$$

Determine η_2

$$\eta_2 = \frac{F_{Ed}}{F_{Rd}} = \frac{V_{Ed}}{F_{Rd}} = \frac{269.5}{325.6} = 0.83 < 1.0$$

Therefore the web resistance to transverse forces is adequate.

2.8 Vertical deflection at serviceability limit state

For this example, the only serviceability limit state that is to be considered is the vertical deflection under variable actions, because excessive deflection would damage brittle finishes which are added after the permanent actions have occurred. The limiting deflection for this beam is taken to be span/360, which is consistent with common design practice.

2.8.1 Design values of combined actions at Serviceability Limit State

As noted in BS EN 1990, the SLS partial factors on actions are taken as unity and expression 6.14a is used to determine design effects. Additionally, as stated in Section 2.2.2, the variable actions are not independent and therefore no combination factors (ψi) are required. Thus, the combination values of actions are given by: BS EN 1990 A1.4.1(1)

$F_{1,d,ser} = g_1 + q_1$ and $F_{2,d,ser} = G_2 + Q_2$

As noted above, the permanent actions considered in this example occur during the construction process, therefore only the variable actions need to be considered in the serviceability verification for the functioning of the structure.

Thus: $F_{1,d,ser} = q_1 = 30.0$ kN/m and $F_{2,d,ser} = Q_2 = 50.0$ kN

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2.8.2 Design value of deflection

The vertical deflection is given by:

$$w = \left(\frac{1}{EI_y}\right)\left(\frac{5F_{1,d,ser}\,L^4}{384} + \frac{F_{2,d,ser}\,L^3}{48}\right)$$

$$= \left(\frac{1}{210000 \times 55200 \times 10^4}\right)\left(\frac{5 \times 30 \times 6500^4}{384} + \frac{50 \times 10^3 \times 6500^3}{48}\right) = 8.48mm$$

The vertical deflection limit is

$$w = \frac{L}{360} = \frac{6500}{360} = 18.1mm$$

8.5 mm < 18.1 mm

Therefore the vertical deflection of the beam is satisfactory.