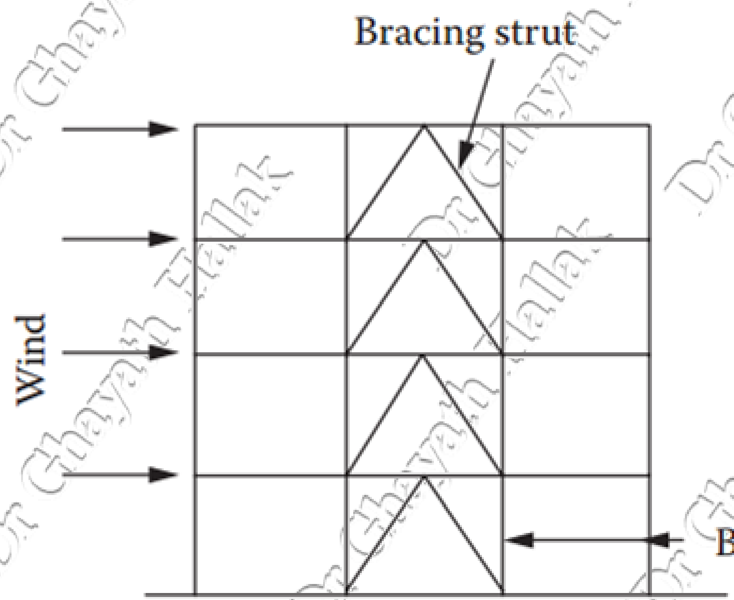
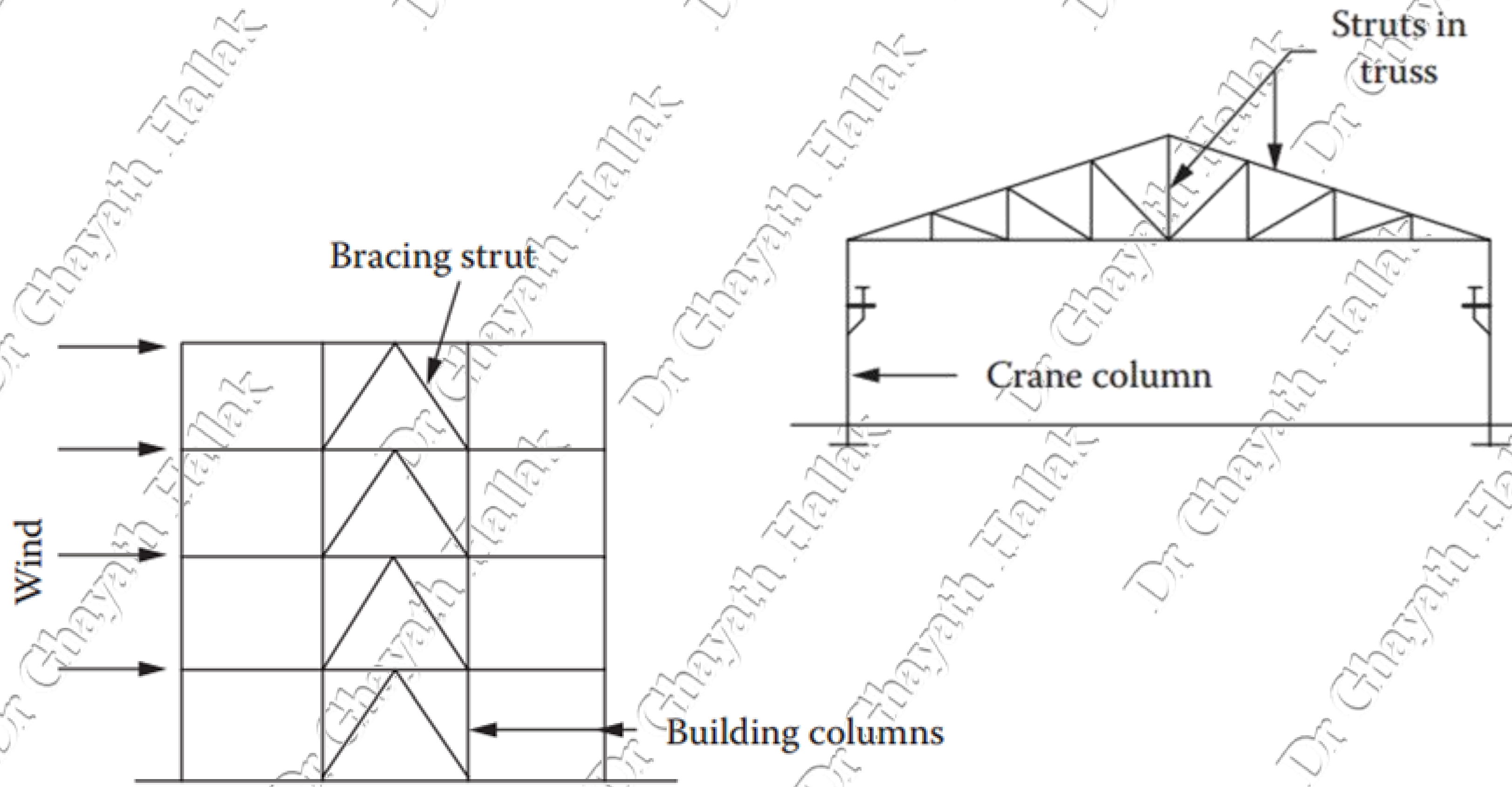
COLUMNS AND STRUTS-(Compression members)

Columns are vertical members supporting floors, roofs and cranes in buildings. Though internal columns in buildings are essentially axially loaded and are designed as such, most columns are subjected to axial load and moment.

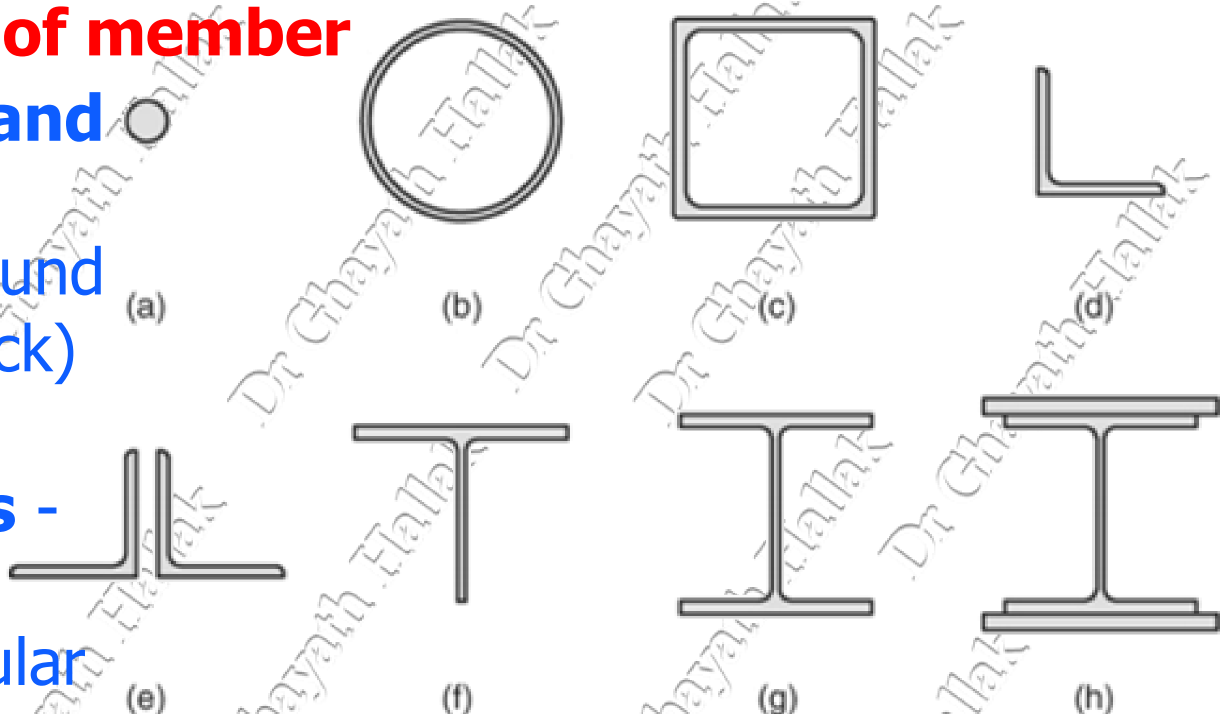


COLUMNS AND STRUTS-(Compression members)

strut is often used to describe other compression members such as those in trusses, lattice girders or bracing.



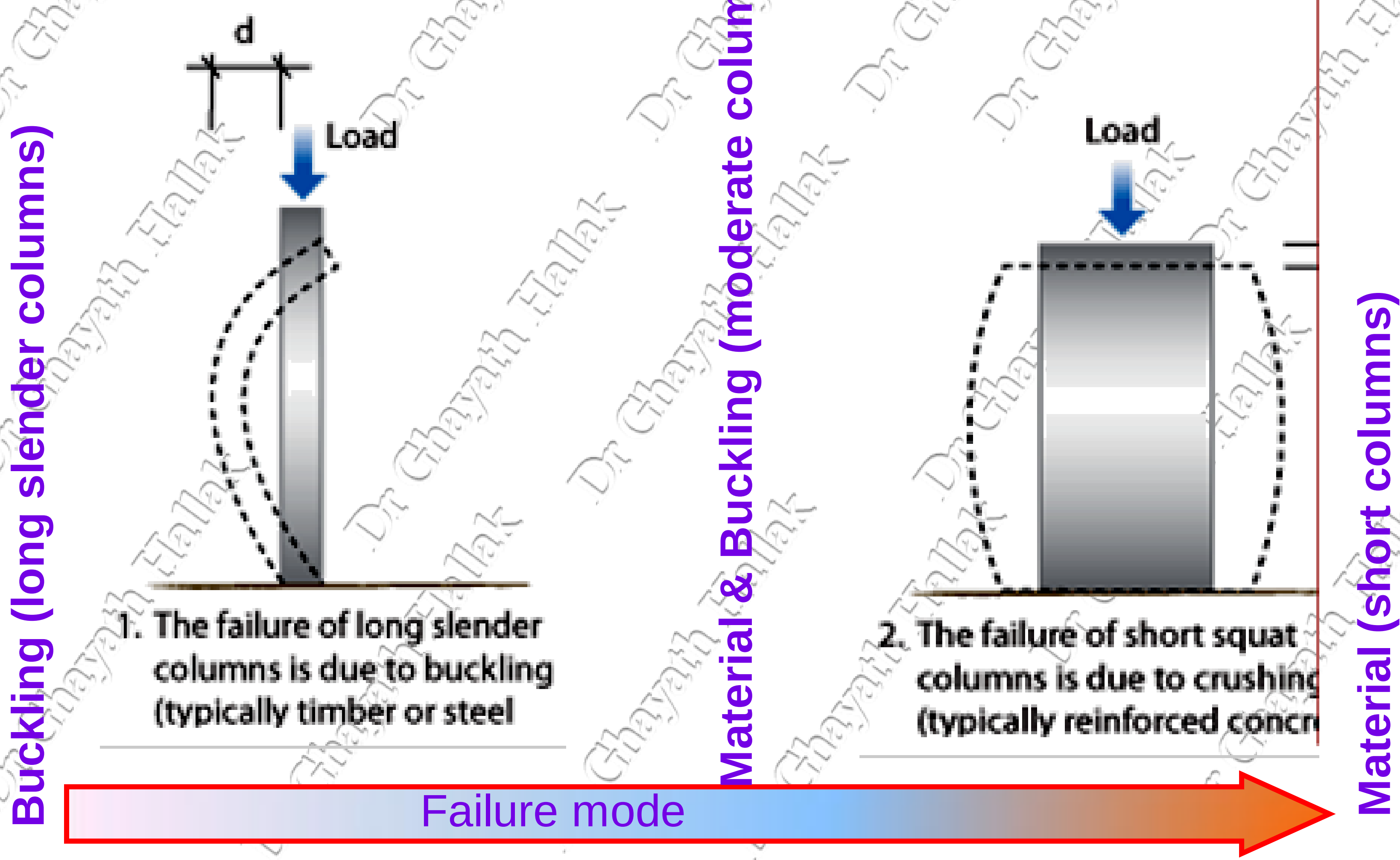
Common types of member



1. **light trusses and bracing** - angles (including compound angles back to back) and tees
2. **larger trusses** - circular hollow sections, rectangular hollow sections, Compound sections and universal columns
3. **frames** - universal columns, fabricated sections e.g. reinforced UCs
4. **bridges** - box columns
5. **power stations** - stiffened box columns.

Design of compression members must take into account

- **material strength**
- **stability against buckling**



EULER THEORY FOR SLENDER COLUMNS

Pin Ended Column

Assumption

1- perfectly straight, 2- homogeneous column. 3- load is applied precisely along the perfectly straight centroidal axis

M, at any section X is then given by

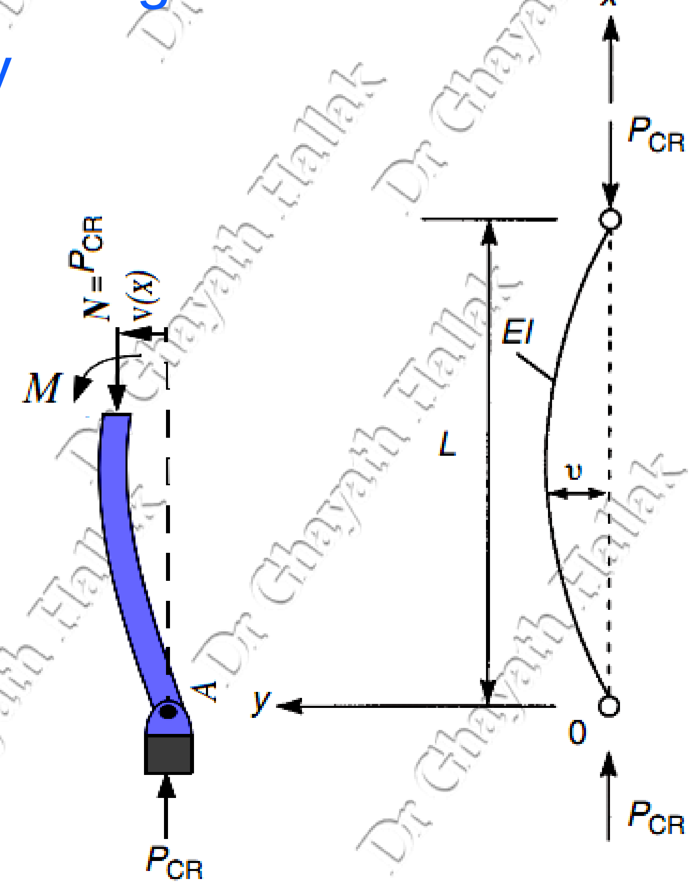
$$M = -P_{CR}\,v$$

$$\frac{d^2v}{dx^2} = \frac{M}{EI} \Rightarrow \frac{d^2v}{dx^2} = -\frac{P_{CR}}{EI}v$$

$$\frac{d^2v}{dx^2} + \frac{P_{CR}}{EI}v = 0$$

$$v = C_1 \cos \mu x + C_2 \sin \mu x$$

$$\Rightarrow P_{CR} = \frac{n^2\pi^2 EI}{L^2}$$

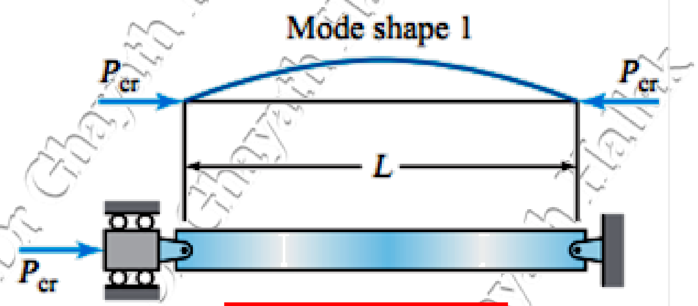


EULER THEORY FOR SLENDER COLUMNS

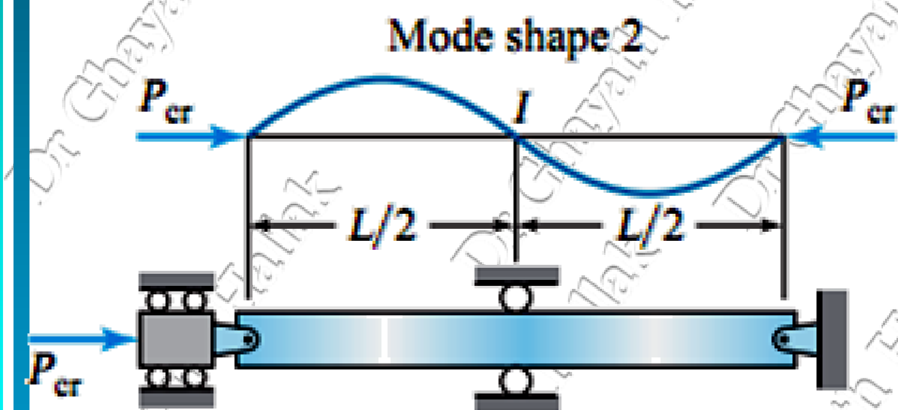
Pin-ended Column

Mode Shape

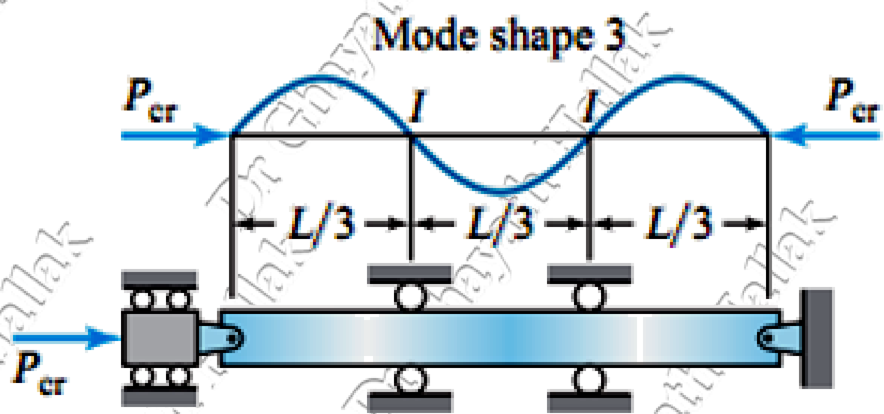
$N_{cr} = P_{cr}$



$$P_{CR} = \frac{\pi^2 EI}{L^2}$$



$$P_{cr} = \frac{4\pi^2 EI}{L^2}$$



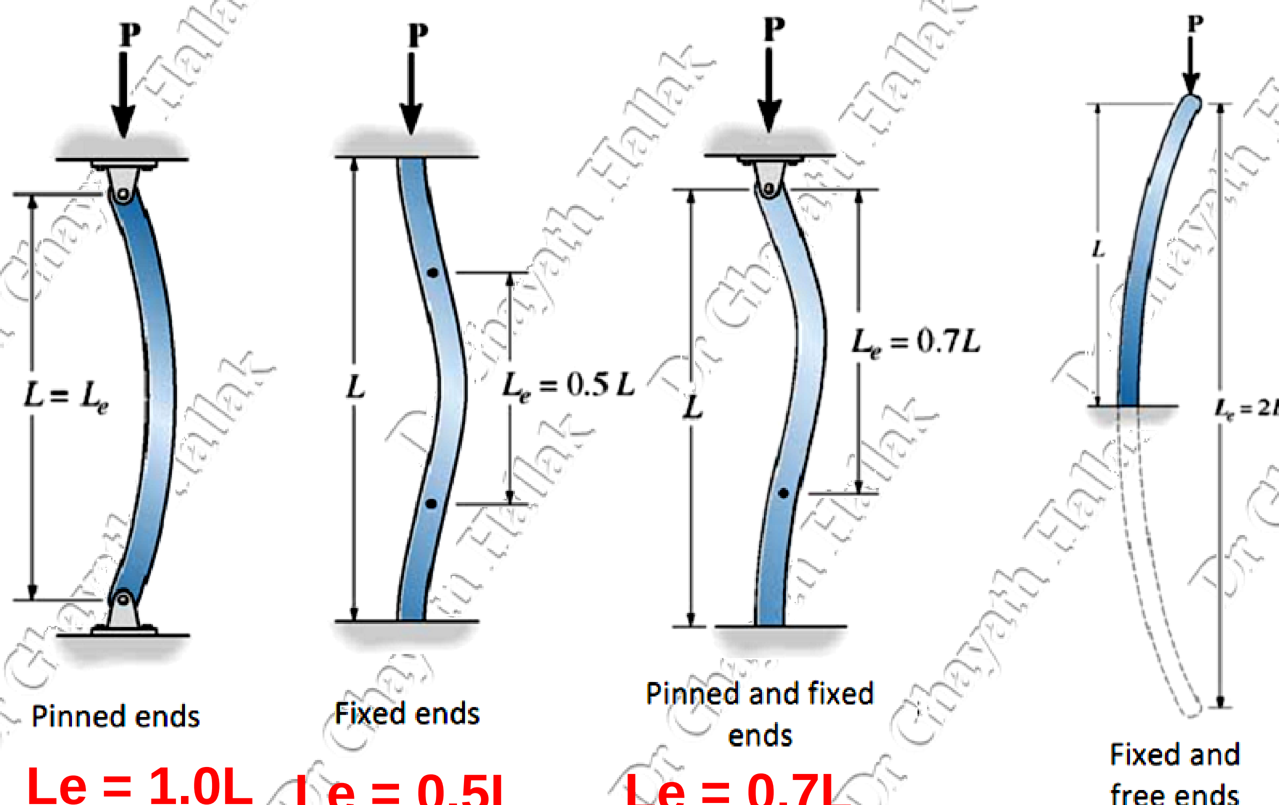
$$P_{cr} = \frac{9\pi^2 EI}{L^2}$$

EULER THEORY FOR SLENDER COLUMNS

Other Boundary Condition Columns

$$N_{cr} = P_{cr} = \frac{\pi^2 EI}{L_E^2}$$

Other boundary conditions may be accounted for through the effective (critical) length concept.



Le = 1.0L Le = 0.5L Le = 0.7L Le = 2.0L

EULER THEORY FOR SLENDER COLUMNS

$$\sigma_{cr} = f_{cr} = \frac{N_{cr}}{A} = \frac{\pi^2 E I}{A L_e^2} = \frac{\pi^2 E i^2}{L_e^2} = \frac{\pi^2 E}{\frac{L_e^2}{i^2}} = \frac{\pi^2 E}{\lambda^2}$$

$$where \quad \lambda = L_e / i \quad slendrness\ ratio$$

$$i = \sqrt{\frac{I}{A}} \quad radius\ of\ gyration$$

For a short member (with a low slenderness ratio), failure occurs by yielding of the cross section

$$\sigma = f_y = \frac{N}{A}$$

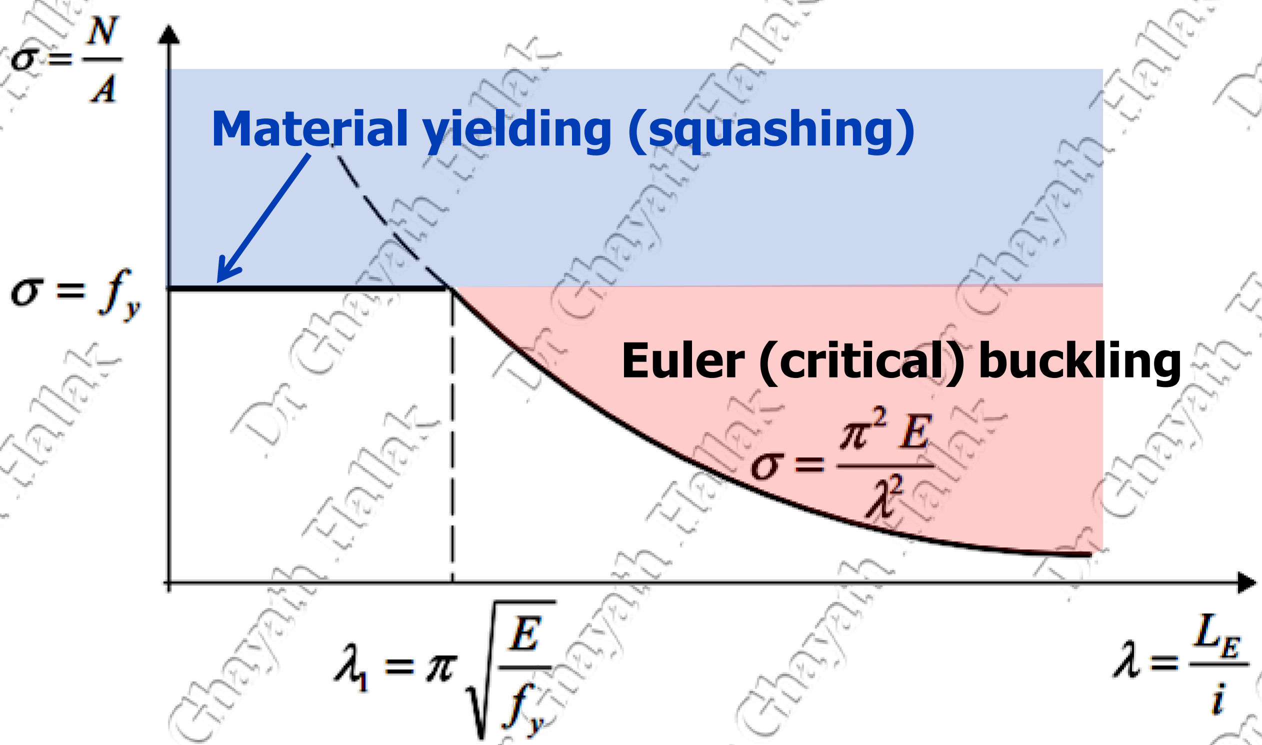
For a slender member (with a high slenderness ratio), failure occurs by buckling of the member

$$\sigma_{cr} = f_{cr} = \frac{\pi^2 E}{\lambda^2}$$

EULER THEORY FOR SLENDER COLUMNS

The limit between the two types of behaviour is defined by a value of the slenderness ratio, denoted as λ_1, given by:

$$\sigma_{cr} = f_y \Rightarrow \frac{\pi^2 E}{\lambda_1^2} = f_y \Rightarrow \lambda_1 = \pi\sqrt{\frac{E}{f_y}}$$



σ- λ relationship of a compressed member

Effect of imperfections and plasticity

In real structures, imperfections are unavoidable and result in deviations from the theoretical behaviour previously described; under these circumstances, the critical load, in general, is not reached.

Imperfections can be divided into two types:

i) geometrical imperfections (lack of linearity, lack of verticality, eccentricity of the loads)

ii) material imperfections (residual stresses).

Effect of imperfections and plasticity

The effect of geometrical imperfections

$$v_0 = a \sin \pi \frac{x}{L}$$

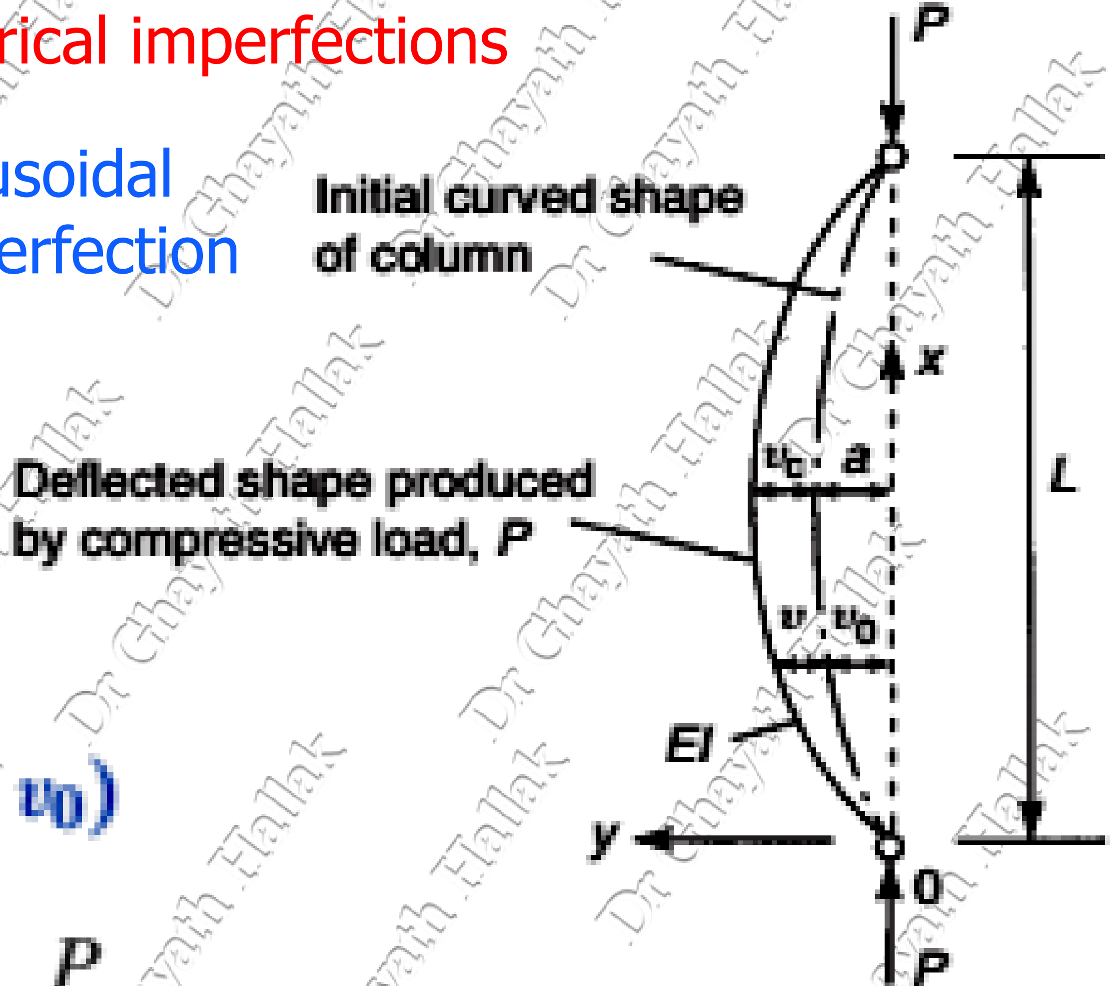
Sinusoidal imperfection

$$M = -P(v_0 + v)$$

$$\frac{d^2 v}{dx^2} = \frac{M}{EI} \Rightarrow$$

$$\frac{d^2 v}{dx^2} = -\frac{P}{EI}(v + v_0)$$

$$\frac{d^2 v}{dx^2} + \frac{P}{EI} v = -\frac{P}{EI} v_0$$



Initial sinusoidal configuration

Effect of imperfections and plasticity

The effect of geometrical imperfections

$$v = C_1 \cos \mu x + C_2 \sin \mu x + \frac{\mu^2 a}{(\pi^2/L^2) - \mu^2} \sin \pi \frac{x}{L}$$

$$\mu^2 = P/EI.$$

If the ends of the column are pinned, $v = 0$ at $x = 0$ and $x = L$.

The first of these boundary conditions gives $C_1 = 0$ while from the second we have

$$0 = C_2 \sin \mu L$$

If $\sin \mu L = 0$ then $\mu L = \pi$ so that $\mu^2 = \pi^2/L^2$. This would then make the third term the above Eq. infinite which is clearly impossible for a column in stable equilibrium ($P < P_{CR}$). We conclude, therefore, that $C_2 = 0$ and hence

Effect of imperfections and plasticity

The effect of geometrical imperfections

$$v = \frac{\mu^2 a}{(\pi^2/L^2) - \mu^2} \sin \pi \frac{x}{L}$$

Dividing the top and bottom of the Eq. by μ^2 we obtain

$$v = \frac{a \sin \pi x/L}{(\pi^2/\mu^2 L^2) - 1} \Rightarrow v = \frac{v_0}{(\pi^2 EI/PL^2) - 1}$$

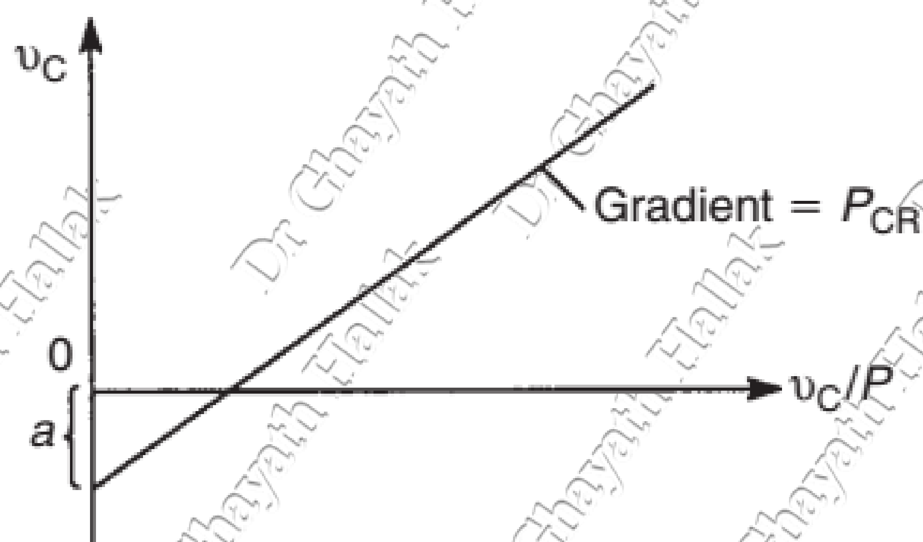
$$\Rightarrow v = \frac{v_0}{(P_{CR}/P) - 1}$$

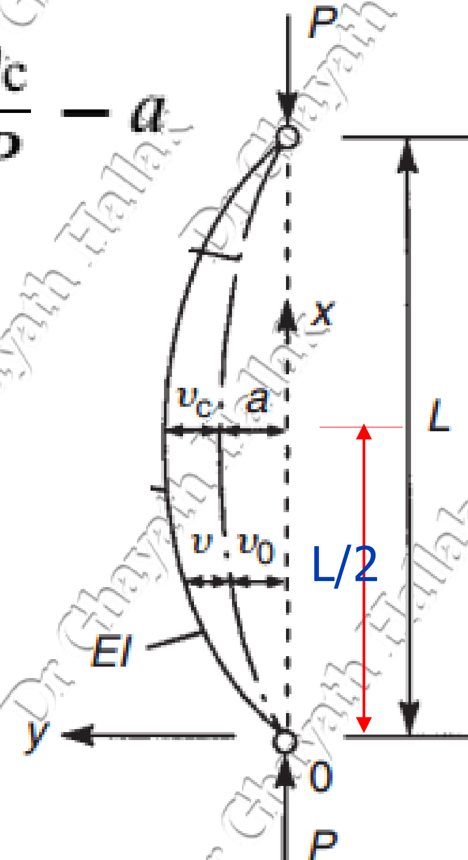
Effect of imperfections and plasticity

The effect of geometrical imperfections

If we consider displacements at the mid-height of the column we have from the previous Eq.

$$v_C = \frac{a}{(P_{CR}/P) - 1} \Rightarrow v_C = P_{CR}\frac{v_C}{P} - a$$





Experimental determination of the buckling load of a column from a Southwell plot

Effect of imperfections and plasticity

The effect of geometrical imperfections

at mid-height $$M_{max} = -P(a + v_c)$$

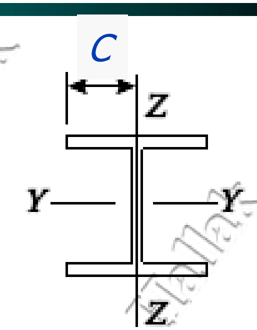
Substituting for v_c: $$M_{max} = -Pa\left(1 + \frac{1}{(P_{CR}/P) - 1}\right)$$

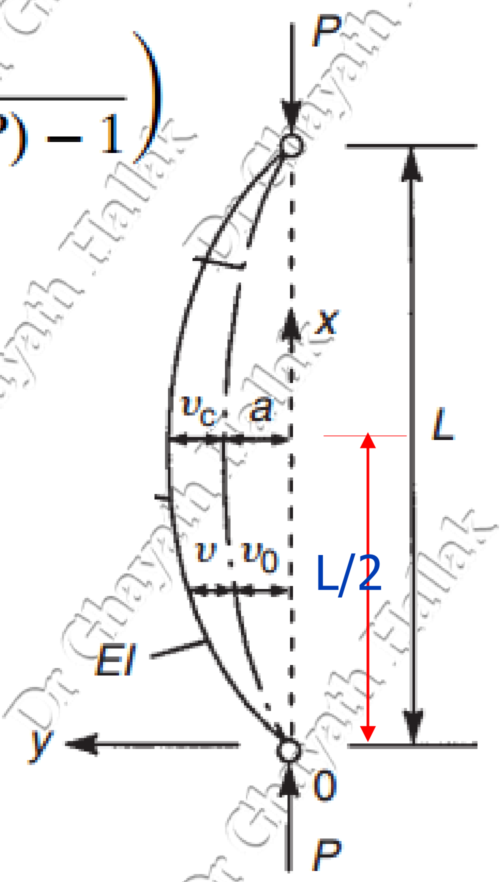
or $$M_{max} = -Pa\left(\frac{P_{CR}}{P_{CR} - P}\right)$$

$$\sigma_{max} = \frac{P}{A} + \frac{M_{max}}{I}c \Rightarrow$$

$$\sigma_{max} = \frac{P}{A} + Pa\left(\frac{P_{CR}}{P_{CR} - P}\right)\left(\frac{c}{I}\right)$$

$$\sigma_{max} = \frac{P}{A} + \frac{P}{A}a\left(\frac{P_{CR}}{P_{CR} - P}\right)\left(\frac{c}{I/A}\right)$$





Effect of imperfections and plasticity

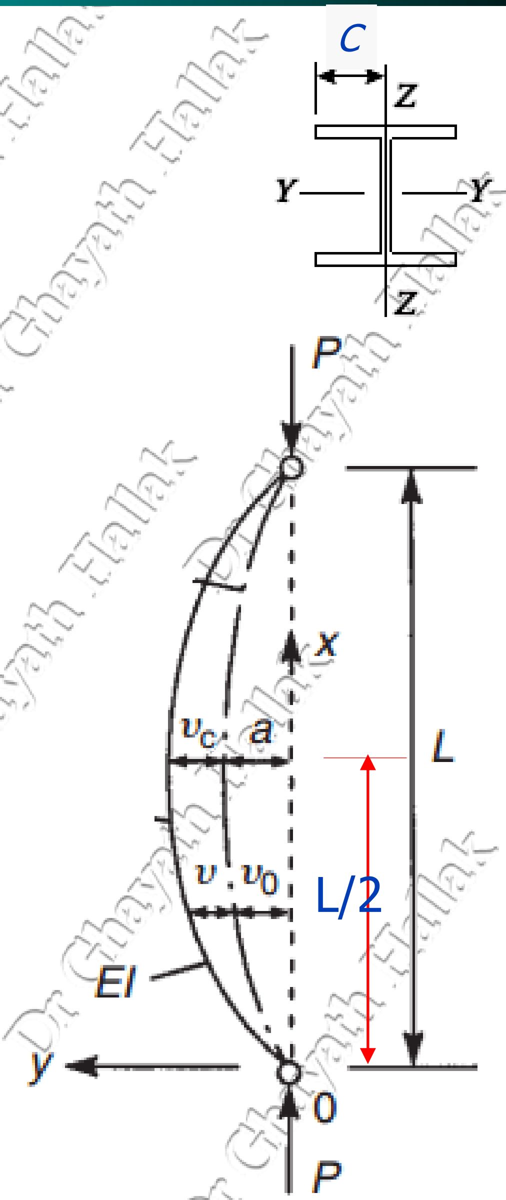
The effect of geometrical imperfections

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{P_{CR}}{P_{CR} - P}\left(\frac{a\,c}{i^2}\right)\right]$$

$$\sigma_{max} = \sigma\left[1 + \frac{\sigma_{CR}}{\sigma_{CR} - \sigma}\left(\frac{a\,c}{i^2}\right)\right]$$

Where $\sigma_{CR} = \frac{P_{CR}}{A} = \frac{\pi^2 EI}{A\,L^2} = \frac{\pi^2 E}{\lambda^2}$

$$\sigma = \frac{P}{A}$$



Effect of imperfections and plasticity

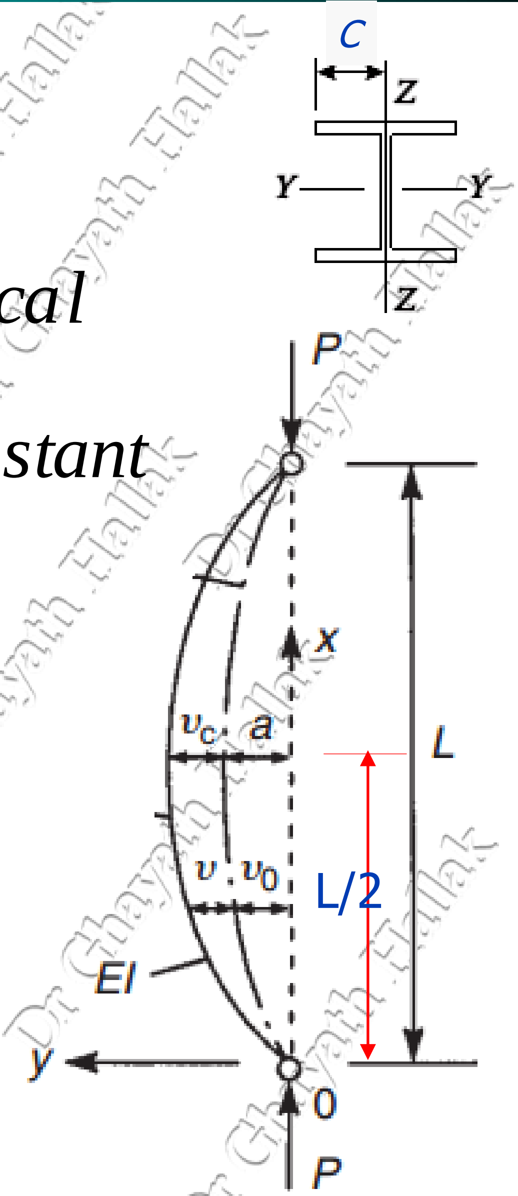
The effect of geometrical imperfections

$$\eta = \frac{a\,c}{i^2}$$ *is an expression of the geometrical configuration of the column and is a constant for a given column having a given initial curvature*

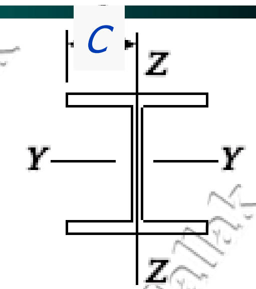
$$\sigma_{\max} = \sigma\left[1 + \frac{\sigma_{CR}}{\sigma_{CR} - \sigma}\eta\right] \Longrightarrow$$

$$\sigma_{\max}\left(\sigma_{CR} - \sigma\right) = \sigma\left[\left(1+\eta\right)\sigma_{CR} - \sigma\right]$$

$$\sigma^2 - \sigma\left[\sigma_{\max} + \left(1+\eta\right)\sigma_{CR}\right] + \sigma_{\max}\sigma_{CR} = 0$$



Effect of imperfections and plasticity



The effect of geometrical imperfections

the solution of which is

$$\sigma = \frac{1}{2}\left[\sigma_{\max} + (1+\eta)\sigma_{CR}\right] - \sqrt{\frac{1}{4}\left[\sigma_{\max} + (1+\eta)\sigma_{CR}\right]^2 - \sigma_{\max}\sigma_{CR}}$$

$$\sigma = \frac{1}{2}\left[\sigma_y + (1+\eta)\sigma_{CR}\right] - \sqrt{\frac{1}{4}\left[\sigma_y + (1+\eta)\sigma_{CR}\right]^2 - \sigma_y\sigma_{CR}}$$

$$\sigma = \frac{1}{2}\left[f_y + (1+\eta)\sigma_{CR}\right] - \sqrt{\frac{1}{4}\left[f_y + (1+\eta)\sigma_{CR}\right]^2 - f_y\sigma_{CR}}$$

the Perry–Robertson formula

The above Equation has been rearranged in the European Code (Eq. (6.49), EN 1993-1-1:2005) to express the buckling stress (σ_{cR}) in terms of the stress ratio ($\sigma_{CR}/f_y = \bar{\lambda}$)

Effect of imperfections and plasticity

The effect of geometrical imperfections

and a reduction factor (χ) related to column imperfections.

If:

$$\zeta = 0.5\left[f_y + (1+\eta)\sigma_{CR}\right]$$

$$\therefore \ \sigma = \zeta - \sqrt{\zeta^2 - f_y\sigma_{CR}}$$

$$\sigma = \left[\zeta - \sqrt{\zeta^2 - f_y\sigma_{CR}}\right]\frac{\left(\zeta + \sqrt{\zeta^2 - f_y\sigma_{CR}}\right)}{\left(\zeta + \sqrt{\zeta^2 - f_y\sigma_{CR}}\right)}$$

$$\sigma = \frac{f_y\sigma_{CR}}{\left(\zeta + \sqrt{\zeta^2 - f_y\sigma_{CR}}\right)}$$

Effect of imperfections and plasticity

The effect of geometrical imperfections

$$\sigma = \frac{f_y}{\left(\frac{\zeta}{\sigma_{CR}} + \sqrt{\left(\frac{\zeta}{\sigma_{CR}}\right)^2 - \frac{f_y}{\sigma_{CR}}}\right)} = \chi f_y$$

In the European Code

$$\phi = \frac{\zeta}{\sigma_{CR}} = 0.5\left[\frac{f_y}{\sigma_{CR}} + (1+\eta)\right], \quad \bar{\lambda} = \frac{f_y}{\sigma_{CR}}$$

$\eta = 0.001\, a\, (\lambda - \lambda_0) > 0$, *where a varies from 2 to 8 depending on the shape of the section and the*

limiting slenderness ratio : $\lambda_0 = 0.2\sqrt{\left(\frac{\pi^2 E}{f_y}\right)} > 0$

Effect of imperfections and plasticity

The effect of geometrical imperfections

$$\phi = 0.5\left[1+\alpha\left(\bar{\lambda}-0.2\right)+\bar{\lambda}^2\right], \alpha = 0.001\,a\sqrt{\frac{\pi^2 E}{f_y}}$$

$$\chi = \frac{1}{\left[\phi+\sqrt{\phi^2-\bar{\lambda}^2}\right]} \leq 1.0$$

cl 6.3.1.2, EN 1993-1-1:2005

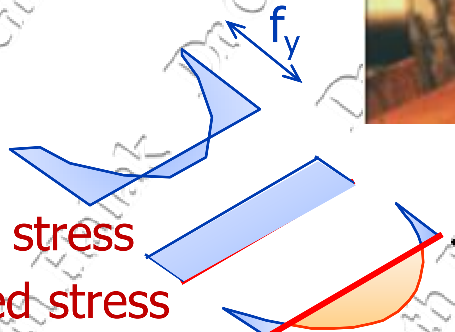
where α is an **imperfection factor**

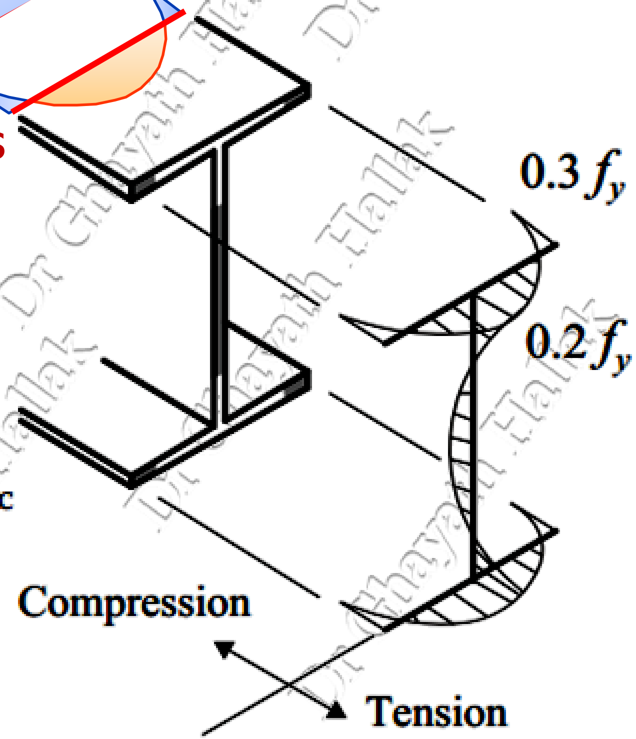
Five different imperfection amplitudes are included (through the imperfection factor α), giving five buckling curves.

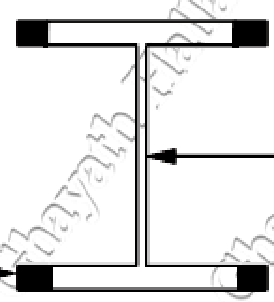
Effect of imperfections and plasticity

The effect of residual stresses

Residual stresses develop due to differential cooling after hot rolling and any other kind of process involving heat (like welding and flame cutting, for example), shearing and cold-forming and cold-bending; despite being a self-equilibrated





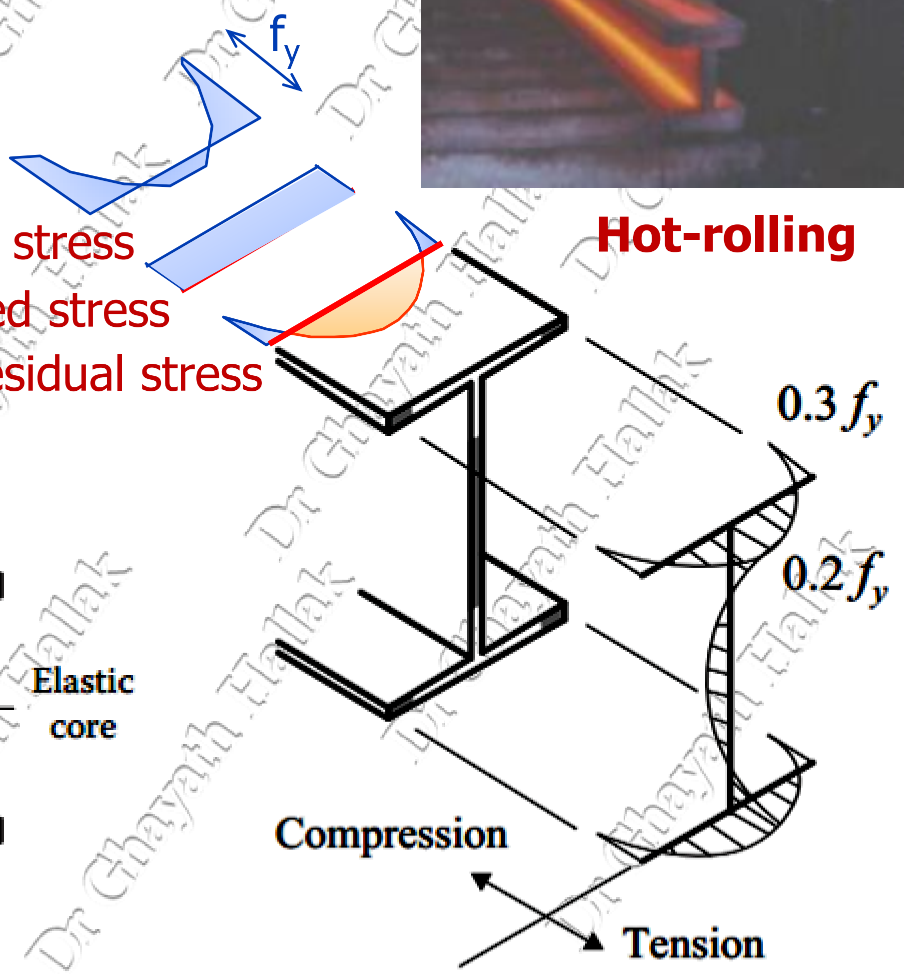


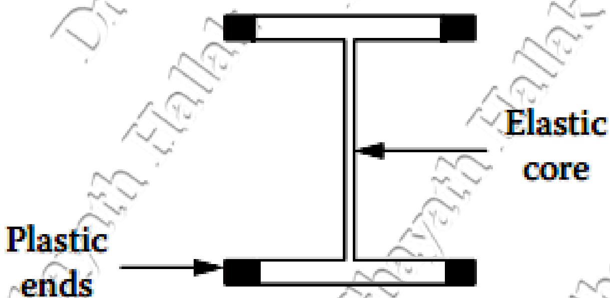
Effect of imperfections and plasticity

The effect of residual stresses

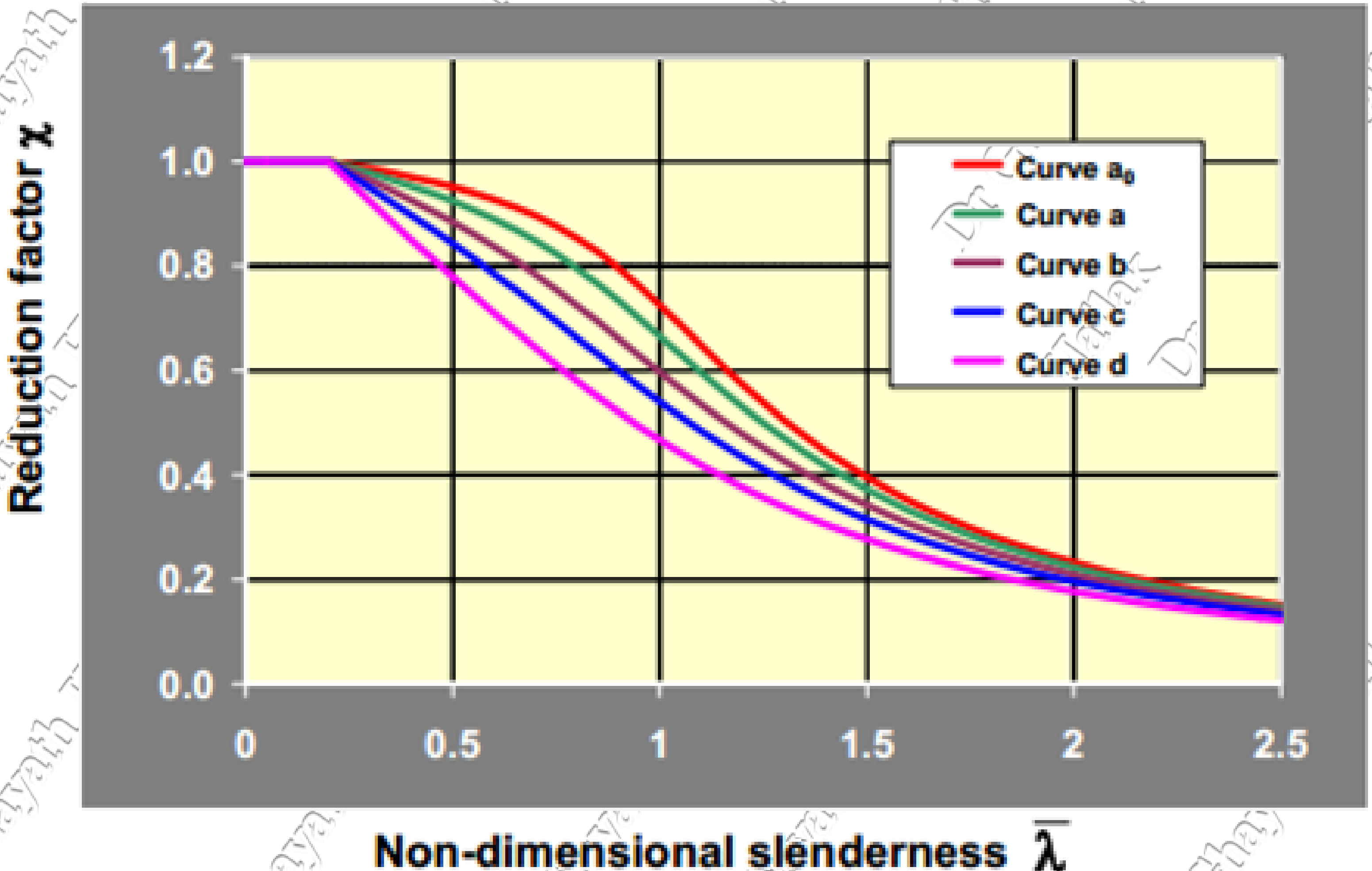
system, these stresses change the load-deformation relationship for the cross section as a whole.

Hot-rolling



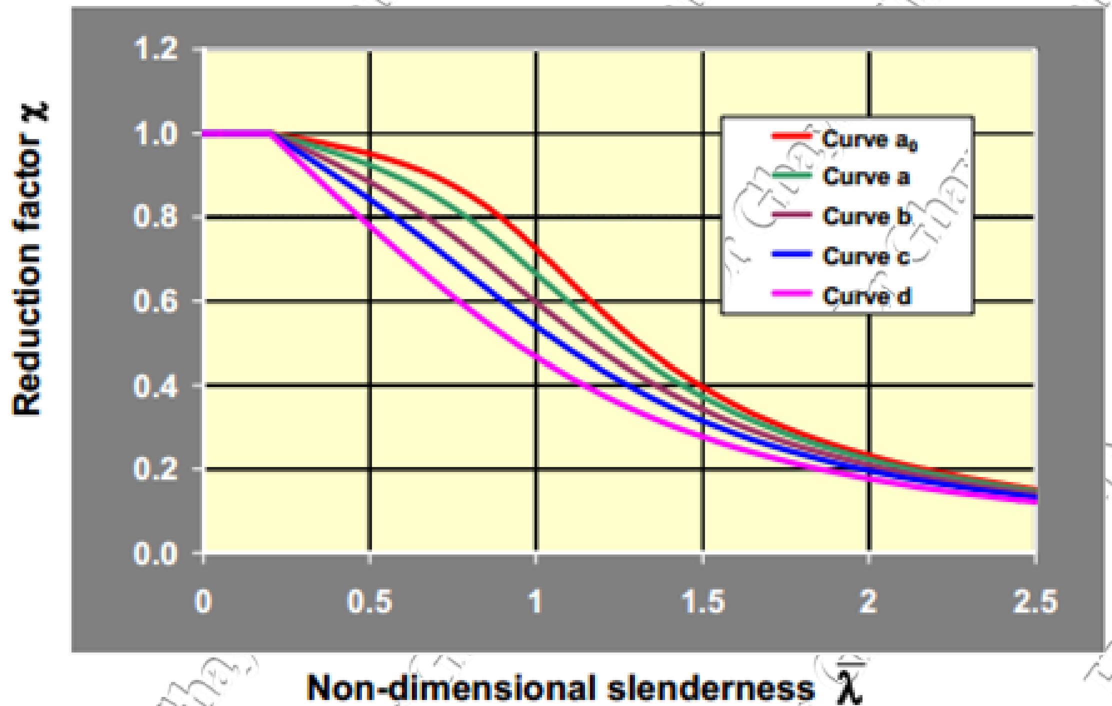


Effect of imperfections and plasticity
Buckling curves Adopted in EN BS 1993-1-1



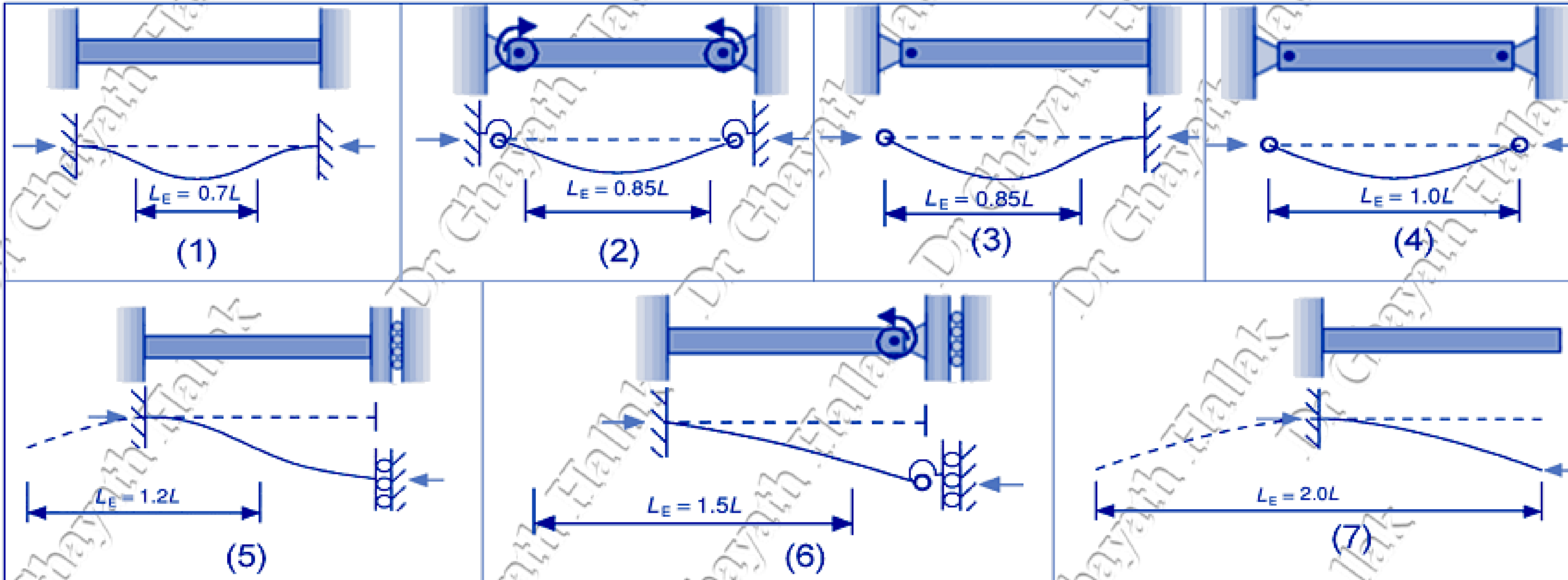
Five curves were the result of an extensive experimental and numerical research programme that

Effect of imperfections and plasticity



accounted for all imperfections in real compressed members (initial out of-straightness, eccentricity of the loads, residual stresses). These imperfections were defined statistically following an extensive measurement campaign that justified the adoption of a sinusoidal geometrical imperfection of amplitude L/1000 in the numerical simulations.

Effective (buckling) lengths $L_{cr}=L_e$ Table 22-BS5950



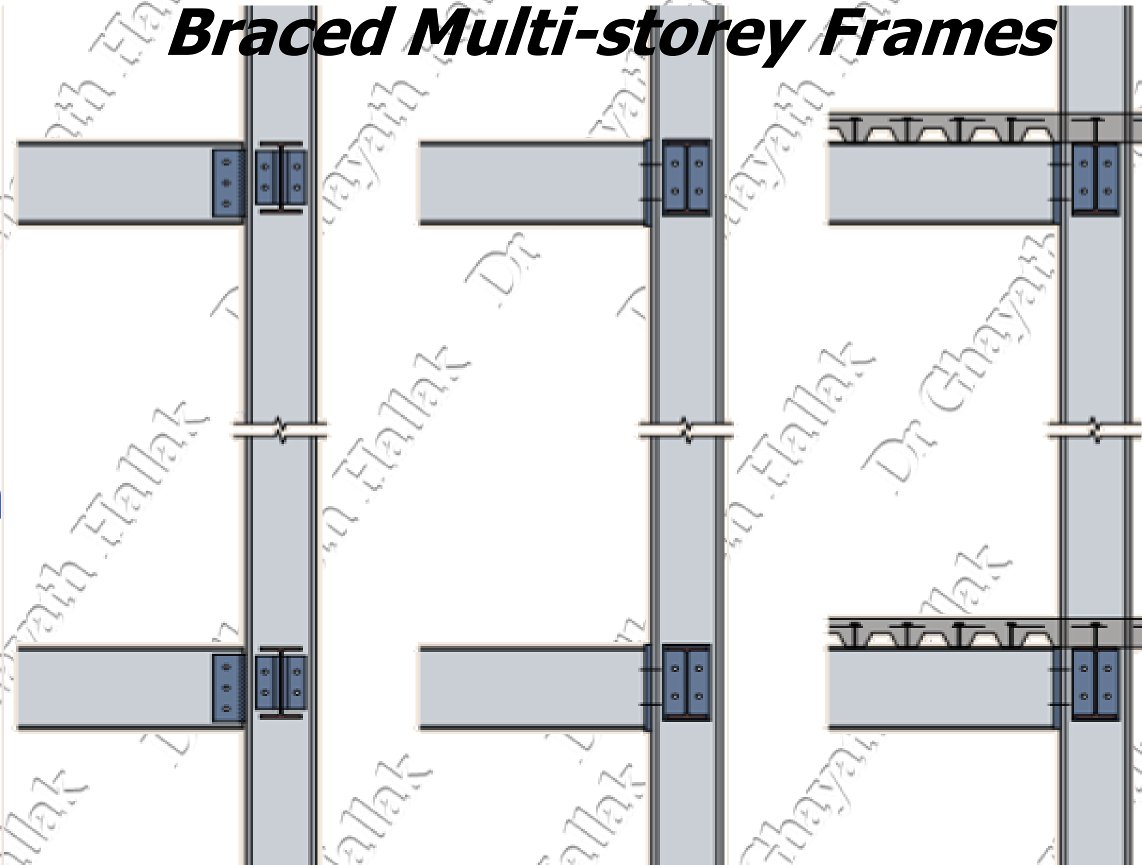
a) non-sway mode				
Restraint (in the plane under consideration) by other parts of the structure				L_E
Effectively held in position at both ends (movement of the ends of a member are restricted)	Effectively restrained in direction at both ends		(1)	0.7L
	Partially restrained in direction at both ends		(2)	0.85L
	Restrained in direction at one end		(3)	0.85L
	Not restrained in direction at either end		(4)	1.0L
b) sway mode				
One end	Other end			L_E
Effectively held in position and restrained in direction	Not held in position	Effectively restrained in direction	(5)	1.2L
		Partially restrained in direction	(6)	1.5L
		Not restrained in direction	(7)	2.0L

Position=Displacement Direction=Rotation

Effective (buckling) lengths $L_{cr}=L_e$ Table 22-BS5950

The rotational end restraint afforded to columns by beam-to-column connections depends on the form of connection and whether the beams are designed compositely.

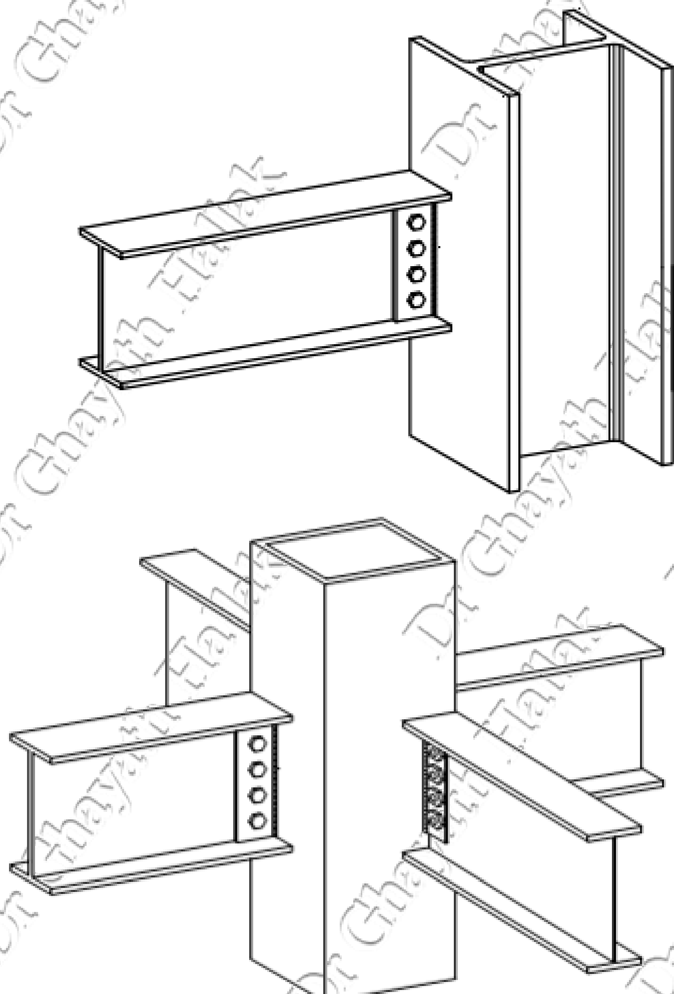
Braced Multi-storey Frames



	a. Fin plate or flexible end plate connections	b. Full depth end plate connections	c. Composite connections
Internal Columns	Lcr= L	Lcr= 0.85L	Lcr= 0.7L
Edge or corner Columns	Lcr= L	Lcr= 0.85L	Lcr= 0.85L

Effective (buckling) lengths $L_{cr}=L_e$ Table 22-BS5950

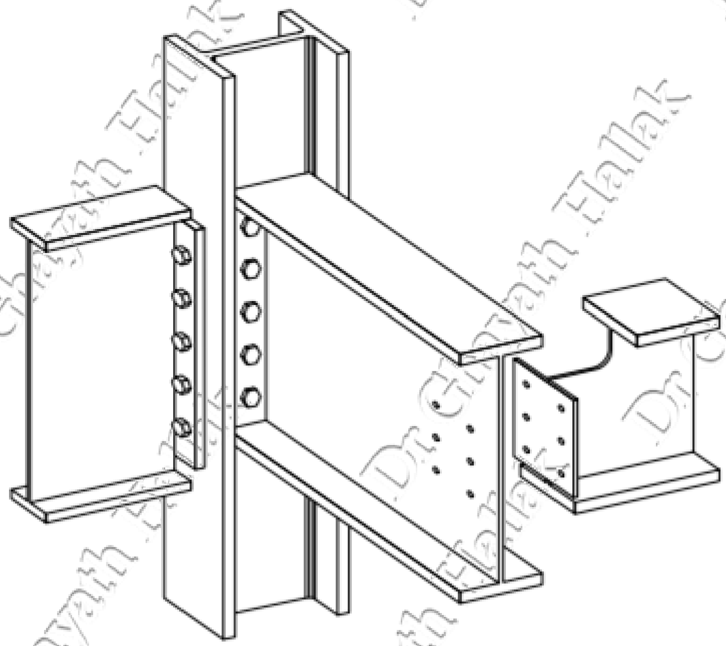
Fin plate connections should not be assumed to provide any rotational restraint.

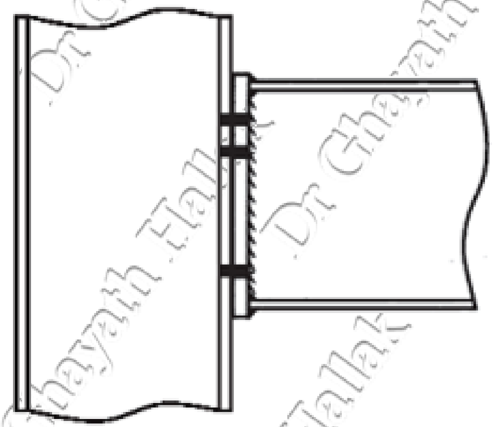


Fin plate beam to column connections

Effective (buckling) lengths $L_{cr}=L_e$ Table 22-BS5950

End plate connections are assumed to offer partial restraint.



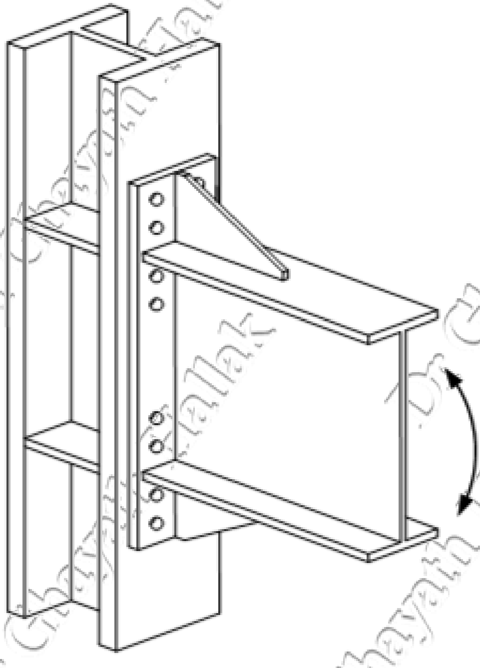


Full depth end plate

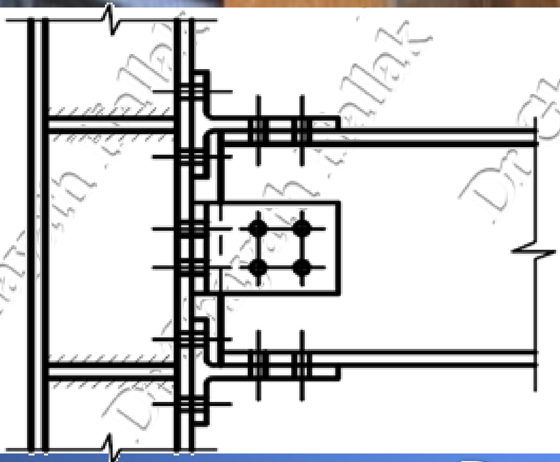
End plate beam to column and beam to beam connections

Effective (buckling) lengths $L_{cr}=L_e$ Table 22-BS5950

Extended End plate connections are assumed to offer full restraint.

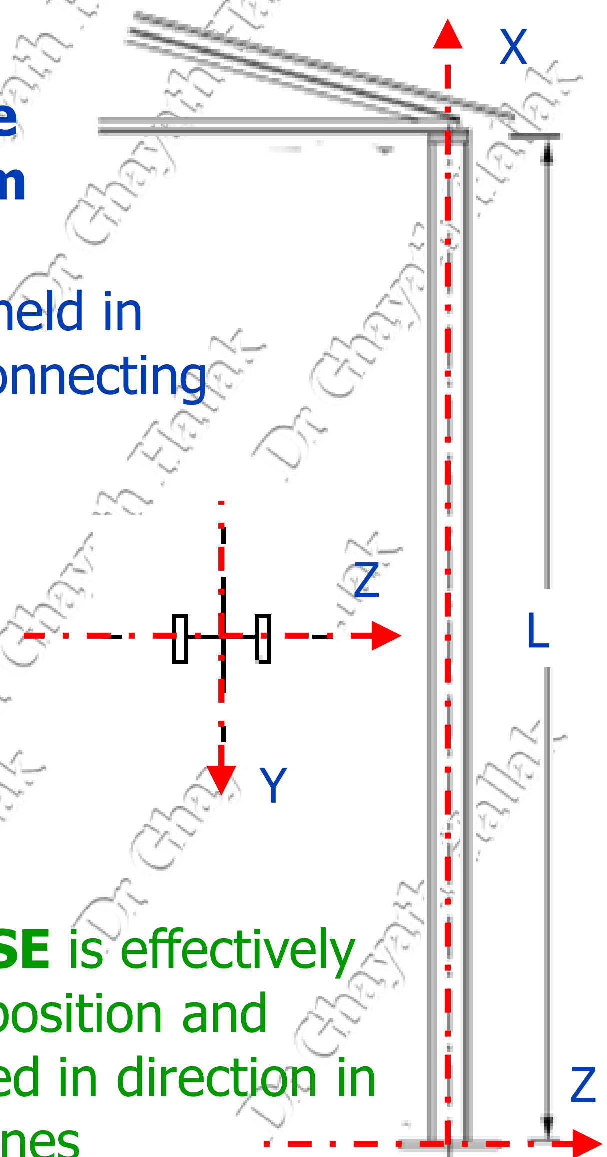


Extended Stiffened End Plate & Column



Effective (buckling) lengths $L_{cr}=L_e$ Annex D-BS5950

Columns for single story buildings

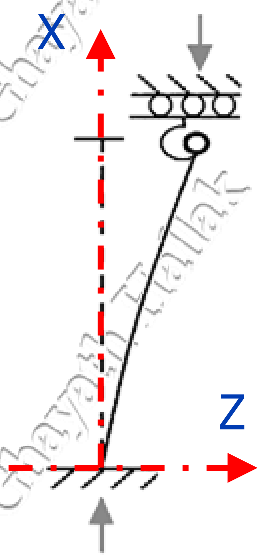


In the plane of the diagram XOZ

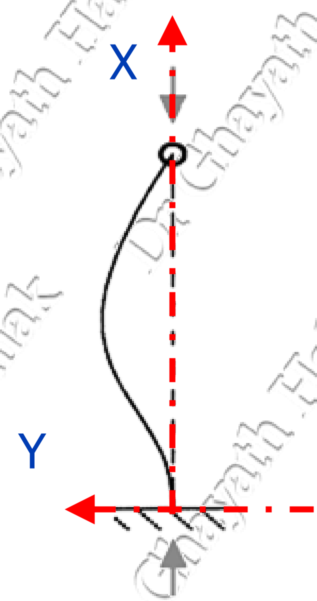
The **TOP** is NOT held in position and it is partially restrained in direction

Perpendicular to the plane of the diagram XOY

The **TOP** is effectively held in position by members connecting them to a braced bay



$L_{Ey} = 1.5L$



$L_{Ez} = 0.85L$

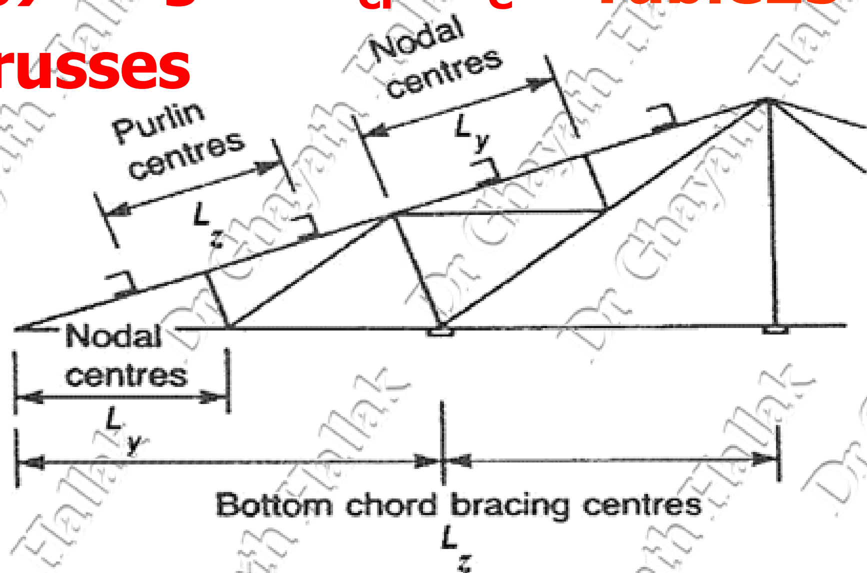
The **BASE** is effectively held in position and restrained in direction in both planes

Effective (buckling) lengths $L_{cr}=L_e$ Table25-BS5950

For Members In Trusses

In-plane buckling:
L_y = joint length

Out-of-plane buckling:
L_z = purlin spacing



end fixities

or

Table25 (BS5950:2000)

$$\lambda_c = \frac{L_v}{i_v} \le 50$$

EN 1993-1-1 Annex BB.1

Table 6.9
$L_v \le 15 i_v$ $\rightarrow$ $\lambda_c = \frac{L_v}{i_v} \le 15$

For single angles

$$\bar{\lambda}_{eff,y} = 0.5 + 0.7\bar{\lambda}_y$$
$$\bar{\lambda}_{eff,z} = 0.5 + 0.7\bar{\lambda}_z$$
$$\bar{\lambda}_{eff,v} = 0.35 + 0.7\bar{\lambda}_v$$

DESIGN FOR COMPRESSION *6.3.1 EN BS 1993-1-1*

Eurocode states that both cross-sectional and member resistance must be verified:

$N_{Ed} \leq N_{c,Rd}$ **Cross-section check**

$N_{Ed} \leq N_{b,Rd}$ **Member buckling check**

Cross-section resistance

❑- Cross-section resistance in compression $N_{c,Rd}$ depends on cross-section classification:

Section classification	Compression
Class 1, 2, 3	$N_{c,Rd}=A\ f_y/\gamma_{M0}$
Class 4	$N_{c,Rd}=A_{eff}\ f_y/\gamma_{M0}$

γ_{M0} is specified as 1.0 in EN 1993

DESIGN FOR COMPRESSION *6.3.1 EN BS 1993-1-1*

Member buckling

Compression buckling resistance $N_{b,Rd}$:

Section classification	Compression
Class 1, 2, 3	$N_{b,Rd} = \chi A f_y / \gamma_{M1}$
Class 4	$N_{b,Rd} = \chi A_{eff} f_y / \gamma_{M1}$

Calculate non-dimensional slenderness $\bar{\lambda}$:

Section classification	non-dimensional slenderness $\bar{\lambda}$
Class 1, 2, 3	$\bar{\lambda} = \sqrt{A f_y / N_{cr}} = \lambda / \lambda_1$
Class 4	$\bar{\lambda} = \sqrt{A_{eff} f_y / N_{cr}} = (\lambda / \lambda_1) \sqrt{A_{eff} / A}$

DESIGN FOR COMPRESSION *6.3.1 EN BS 1993-1-1*

Calculate non-dimensional slenderness λ:

$$N_{cr} = \frac{\pi^2 E I}{L_e^2}$$

$$\sigma_{cr} = f_{cr} = \frac{N_{cr}}{A} = \frac{\pi^2 E I}{A L_e^2} = \frac{\pi^2 E i^2}{L_e^2} = \frac{\pi^2 E}{\frac{L_e^2}{i^2}} = \frac{\pi^2 E}{\lambda^2}$$

$$where\ \lambda = L_e / i \quad slendrness\ ratio, i = \sqrt{I/A} \quad radius\ of\ gyration$$

The limit between the material yielding (short columns) and elastic member buckling (slender columns) is defined by a value of the slenderness ratio, denoted as λ_1, given by:

$$\sigma_{cr} = f_y \Rightarrow \frac{\pi^2 E}{\lambda_1^2} = f_y \Rightarrow \lambda_1 = \pi\sqrt{\frac{E}{f_y}} = \pi\sqrt{\frac{210000}{f_y} \times \frac{235}{235}}$$

$$\lambda_1 = 93.9\sqrt{235/f_y} = 93.9\,\varepsilon$$

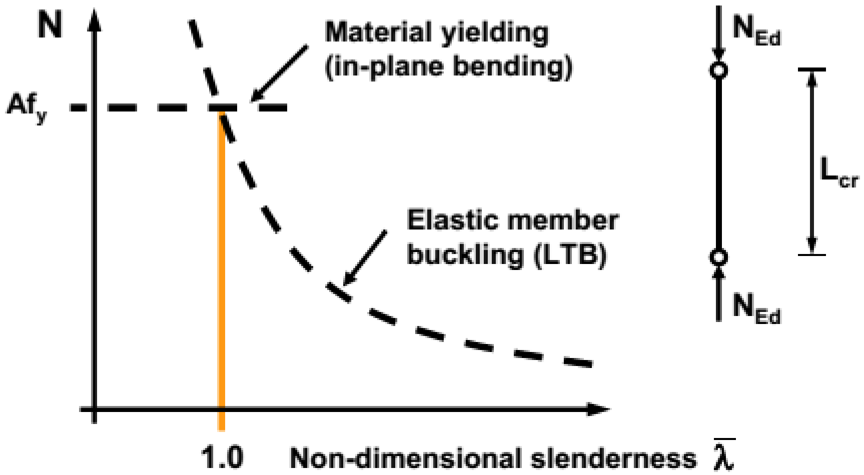
DESIGN FOR COMPRESSION *6.3.1 EN BS 1993-1-1*

Member buckling

Calculate non-dimensional slenderness $\bar{\lambda}$:

The non-dimensional slenderness used in EN 1993-1-1 is defined as:

$$\bar{\lambda} = \frac{\lambda}{\lambda_1} = \frac{\left(\frac{L_e}{i}\right)}{\pi\sqrt{\frac{E}{f_y}}} = \frac{\sqrt{\frac{\pi^2 E}{f_{cr}}}}{\pi\sqrt{\frac{E}{f_y}}} = \frac{\sqrt{\frac{1}{f_{cr}}}}{\sqrt{\frac{1}{f_y}}} = \sqrt{\frac{f_y}{f_{cr}}} = \sqrt{\frac{Af_y}{N_{cr}}} = \frac{\left(\frac{L_e}{i}\right)}{93.9\,\varepsilon}$$



DESIGN FOR COMPRESSION *6.3.1 EN BS 1993-1-1*
Member buckling

Calculate reduction factor, χ

$$\chi = \frac{1}{\left[\phi + \sqrt{\phi^2 - \bar{\lambda}^2}\right]} \leq 1.0, \quad \phi = 0.5\left[1 + \alpha\left(\bar{\lambda} - 0.2\right) + \bar{\lambda}^2\right]$$

α is the imperfection factor

Table 6.1: Imperfection factors for buckling curves *EN 1993-1-1*

Buckling curve	a_0	a	b	c	d
Imperfection factor α	0,13	0,21	0,34	0,49	0,76

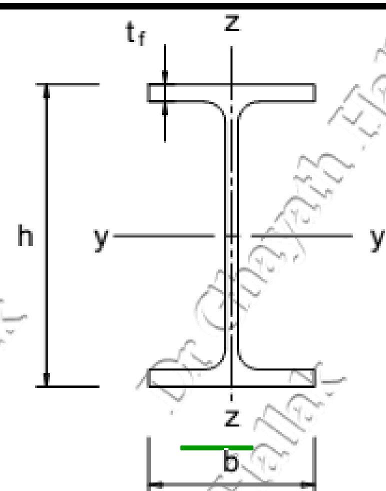
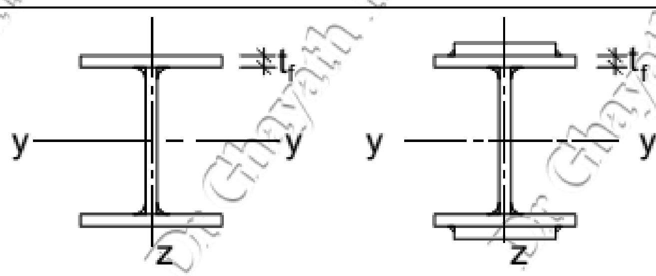
For slenderness $\bar{\lambda} \leq 0.2$ or $N_{Ed}/N_{cr} \leq 0.4$ the buckling effects may be ignored and only cross sectional checks apply.

DESIGN FOR COMPRESSION *6.3.1 EN BS 1993-1-1*

Member buckling

Buckling curve selection

Table 6.2: Selection of buckling curve for a cross-section *EN 1993*

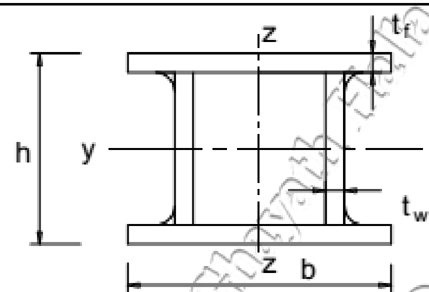
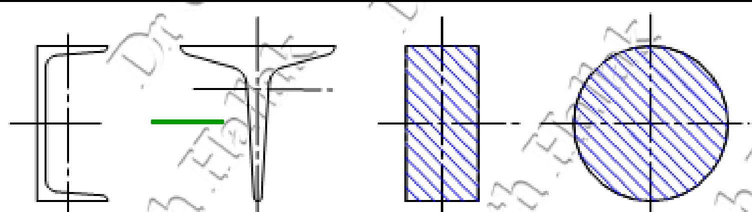
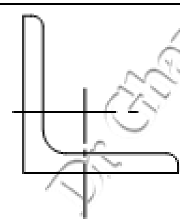
Cross section		Limits		Buckling about axis	Buckling curve: S 235, S 275, S 355, S 420	Buckling curve: S 460
Rolled sections		$h/b > 1,2$	$t_f \le 40$ mm	y – y	a	a_0
				z – z	b	a_0
			40 mm $< t_f \le 100$	y – y	b	a
				z – z	c	a
		$h/b \le 1,2$	$t_f \le 100$ mm	y – y	b	a
				z – z	c	a
			$t_f > 100$ mm	y – y	d	c
				z – z	d	c
Welded I-sections		$t_f \le 40$ mm		y – y	b	b
				z – z	c	c
		$t_f > 40$ mm		y – y	c	c
				z – z	d	d

DESIGN FOR COMPRESSION *6.3.1 EN BS 1993-1-1*

Member buckling

Buckling curve selection

Table 6.2: Selection of buckling curve for a cross-section

Hollow sections		hot finished	any	a	a_0
		cold formed	any	c	c
Welded box sections		generally (except as below)	any	b	b
		thick welds: $a > 0,5t_f$ $b/t_f < 30$ $h/t_w < 30$	any	c	c
U-, T- and solid sections			any	c	c
L-sections			any	b	b

DESIGN FOR COMPRESSION *6.3.1 EN BS 1993-1-1*

Design procedure for column buckling:

1. Determine design axial load N_{Ed}
2. Select section and determine geometry
3. Classify cross-section (if Class 1-3, no account need be made for local buckling)
4. Determine effective (buckling) length L_{cr}
5. Calculate N_{cr} and Af_y
6. Non-dimensional slenderness $\bar{\lambda} = \sqrt{\frac{Af_y}{N_{cr}}}$
7. Determine imperfection factor α
8. Calculate buckling reduction factor χ
9. Design buckling resistance $\mathbf{N_{b,Rd} = \chi A f_y / \gamma_{M1}}$
10. Check $\mathbf{N_{ed} / N_{b,Rd} \leq 1.0}$

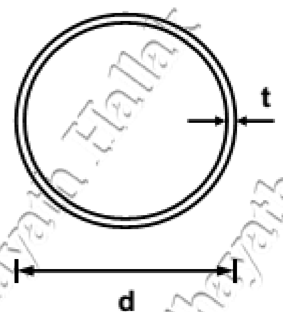
Example

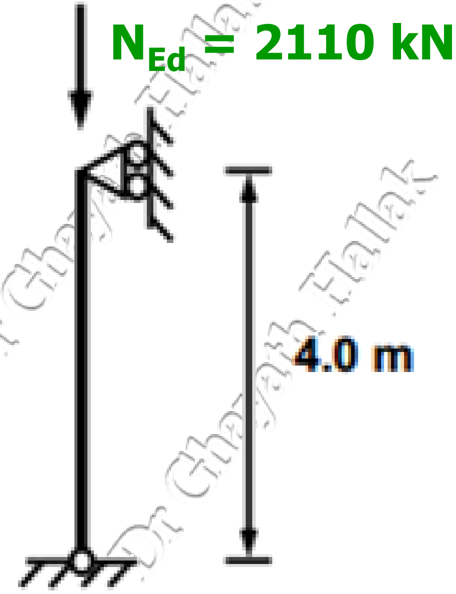
A circular hollow hot-rolled section 244.5×10 CHS member is to be used as an internal column in a multi-storey building. The column has pinned boundary conditions at each end, and the inter-storey height is 4 m. The critical combination of actions results in a design axial force of 2110kN. Verify the validity of this member.

Use grade S 355 steel.

Section Specification

- $d = 244.5$ mm
- $t = 10.0$ mm
- $A = 7370$ mm2
- $W_{el,y} = 415000$ mm3
- $W_{pl,y} = 550000$ mm3
- $I = 50730000$ mm4





Example

Material Specification

t = 10.0 mm>16mm ⇨ f_y=355Mpa=355N/mm^2

E = 210000 N/mm^2

Cross-section classification (*clause 5.5.2*):

$\varepsilon = (235/f_y)^{0.5} = (235/355)^{0.5} = 0.81$

Tubular sections (*Table 5.2, sheet 3*):

d/t = 244.5/10.0 = 24.5

Limit for Class 1 section = 50 ε^2 = 40.7 > 24.5

∴ Cross-section is Class 1

Cross-section compression resistance (clause 6.2.4):

$N_{c,Rd} = A\, f_y/\gamma_{M0}$ = [(7370x355)/1.0]x10^{-3}=2616kN>2110kN

∴ Cross section resistance is OK

Example

Member buckling resistance in compression (clause 6.3.1):

$$N_{b,Rd} = \chi A f_y/\gamma_{M1} \quad , \qquad \bar{\lambda} = \frac{\lambda}{\lambda_1} = \frac{\left(\dfrac{L_e}{i}\right)}{93.9\,\varepsilon} = \sqrt{\frac{Af_y}{N_{cr}}}$$

ε =0.81, Le=4m, i=(I/A)$^{0.5}$=(50730000/7370)$^{0.5}$=82.96mm

$$\bar{\lambda} = \frac{\lambda}{\lambda_1} = \frac{\left(\dfrac{L_e}{i}\right)}{93.9\,\varepsilon} = \frac{\left(\dfrac{4000}{82.96}\right)}{93.9 \times 0.81} = 0.63$$

$$N_{cr} = \frac{\pi^2 E I}{L_e^2} = \frac{\pi^2 \times 210000 \times 50730000}{4000^2} = 6564830\ N$$

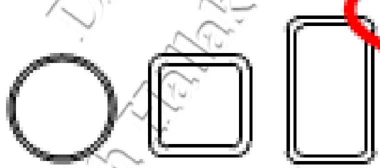
$$\bar{\lambda} = \sqrt{\frac{Af_y}{N_{cr}}} = \sqrt{\frac{7370 \times 355}{6564830}} = 0.63$$

Example

Member buckling resistance in compression (clause 6.3.1):

From Table 6.2 of EN 1993-1-1:

For a hot-rolled CHS, use buckling curve **a**

Cross-section		Limits	Buckling about axis	Buckling curve	
				S235 S275 S355 S420	S460
Hollow sections		hot finished	any	a	a_0
		cold formed	any	c	c

Example

Member buckling resistance in compression (clause 6.3.1):

From Table 6.1 of EN 1993-1-1, for buckling curve a, α=0.21

$$\phi = 0.5\left[1 + \alpha\left(\bar{\lambda} - 0.2\right) + \bar{\lambda}^2\right]$$

$$\phi = 0.5\left[1 + 0.21 \times \left(0.63 - 0.2\right) + 0.63^2\right] = 0.74$$

$$\chi = \frac{1}{\left[\phi + \sqrt{\phi^2 - \bar{\lambda}^2}\right]} = \frac{1}{\left[0.75 + \sqrt{0.75^2 - 0.63^2}\right]} = 0.88$$

$N_{b,Rd} = \chi A f_y / \gamma_{M1} = 0.88 \text{x} 7370 \text{x} 355/1.0 = 2297 \text{x} 10^3 N = 2297 kN$

2297 kN > 2110 kN ∴ Buckling resistance is OK.

The chosen cross-section, 244.5x10 CHS, in grade S 355 steel is acceptable.

Example: Built-up column

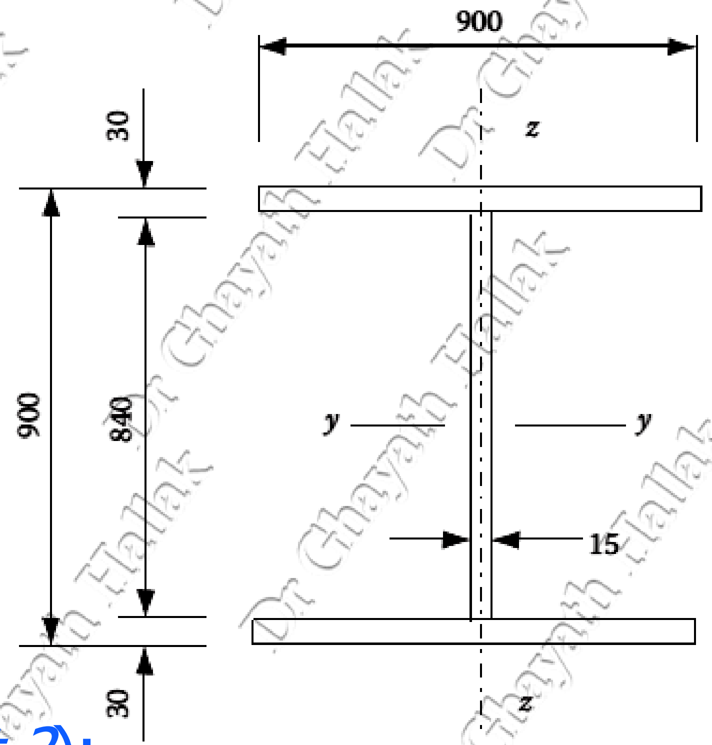
Determine the compression resistance of the column section shown in the Figure. The weld size is 8 mm. The buckling length of the column is 8 m and the steel is S275.

Material Specification

40mm>t_{max} = 30.0 mm>16mm

⇨ f_y=265Mpa=265N/mm^2

E = 210000 N/mm^2



Cross-section classification (*clause 5.5.2*):

$\varepsilon = (235/f_y)^{0.5} = (235/265)^{0.5} = 0.94$

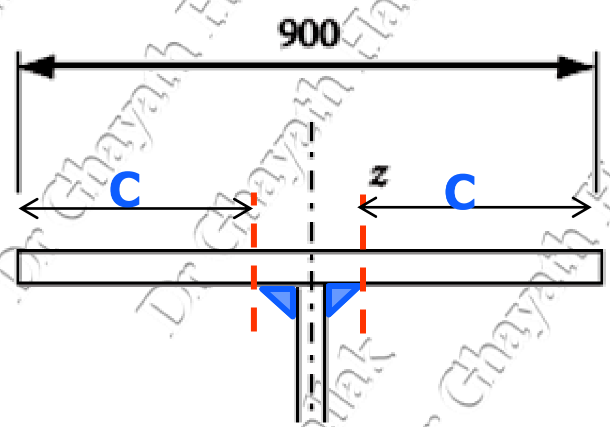
. Flanges

outstand flange (*Table 5.2, sheet 2*):

c/t_f = [(900/2 – 15/2) – 8]/30 = 14.48 ≥ 14ε Class 4

Example: Built-up column

Cross-section classification (*clause 5.5.2*):



From BS EN1993-1-5, Table 4.2,
k_σ = 0.43

$$\bar{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{cr}}} = \frac{\bar{b}/t}{28.4\,\varepsilon\,\sqrt{k_\sigma}}$$

$$\bar{\lambda}_p = \frac{14.48}{28.4 \times 0.94 \times \sqrt{0.43}} = 0.827$$

$$for\ \bar{\lambda}_p > 0.748\ ,\ \rho = \frac{\bar{\lambda}_p - 0.188}{\bar{\lambda}_p^2} \le 1.0,\ \rho = \frac{0.827 - 0.188}{0.827^2} = 0.93$$

$A_{eff,f}$ = 0.93 × 4 × (900/2 − 15/2 − 8) × 30 + (15 + 2 × 8) × 30 × 2 = 49829 mm2

Example: Built-up column

Cross-section classification

. Web

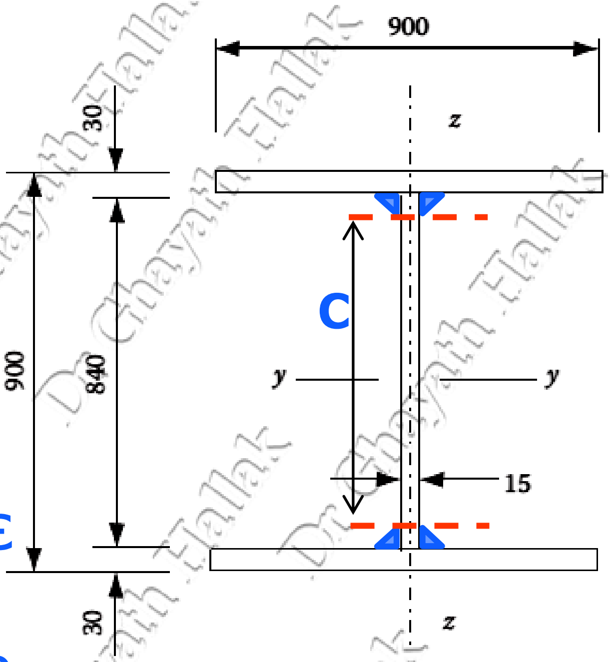
This is an internal element in axial compression, *Table 5.2, sheet 2* :

t_w = 15 mm, ε = 0.94

c/t_w = (840 – 2 × 8)/15 = 54.93 ≥ 42ε

Class 4

From BS EN1993-1-5, Table 4.1, k_σ = 4.0



$$\bar{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{cr}}} = \frac{\bar{c}/t}{28.4\varepsilon\sqrt{k_\sigma}} = \frac{54.93}{28.4 \times 0.94 \times \sqrt{4}} = 1.029$$

$$for\ \bar{\lambda}_p > 0.673, \rho = \frac{\bar{\lambda}_p - 0.055(3+\psi)}{\bar{\lambda}_p^2} = 0.76$$

$A_{eff,w}$ = 0.76 × (840-16) × 15+8 ×15 ×2 = 9634 mm2

Example: Built-up column

3. Properties of the gross section and effective section

Gross area = (2 × 30 × 900) + (840 × 15) = 66 600 mm^2

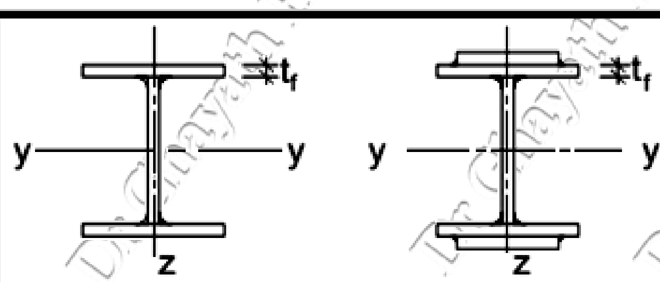
I_z =2×(30 × 900^3/12)+(840 × 15^3/12)=3.645×10^{9}mm^4

i_z = [3.645 × 109/6.66 × 10^4]$^{0.5}$ = 233.9 mm

$\lambda_z = L_{cr} / i_z$ = 8000/233.9 = 34.2

Effective sectional area = 49829 + 9634 = 59463 mm^2.

4. Compressive resistance of the column From Table 6.2 of EN1993-1-1, for an S275 welded I section, t_f less than 40mm, buckling about the minor *z–z* axis, use buckling curve **c**

Cross section		Limits	Buckling about axis	Buckling curve: S 235, S 275, S 355, S 420	Buckling curve: S 460
Welded I-sections		$t_f \leq 40$ mm	y – y z – z	b c	b c
		$t_f > 40$ mm	y – y z – z	c d	c d

Example: Built-up column

From Table 6.1 of EN 1993-1-1, for buckling curve C, imperfection factor, α = 0.49

Buckling curve	a_0	a	b	c	d
Imperfection factor α	0,13	0,21	0,34	0,49	0,76

$$\bar{\lambda} = \frac{\lambda}{\lambda_1} \times \sqrt{\frac{A_{eff}}{A}} = \frac{\left(\frac{L_e}{i}\right)}{93.9\,\varepsilon} \times \sqrt{\frac{A_{eff}}{A}} = \frac{\left(\frac{8000}{233.9}\right)}{93.9 \times 0.94} \times \sqrt{\frac{59463}{66600}}$$

$$\bar{\lambda} = 0.37$$

$$\phi = 0.5\left[1 + \alpha\left(\bar{\lambda} - 0.2\right) + \bar{\lambda}^2\right]$$

$$\phi = 0.5\left[1 + 0.49 \times (0.37 - 0.2) + 0.37^2\right] = 0.61$$

$$\chi = \frac{1}{\left[\phi + \sqrt{\phi^2 - \bar{\lambda}^2}\right]} = \frac{1}{\left[0.61 + \sqrt{0.61^2 - 0.37^2}\right]} = 0.91$$

Example: Built-up column

$$N_{b,Rd} = \chi\, A_{eff}\, f_y/\gamma_{M1} = 0.91 \times 59463 \times 265/1.0 = 14340 \times 10^3 N$$

$$= 14340\ kN$$