Faculty of Civil Engineering CALCULATION SHEET	Job No.	Sheet 1 of 8	Rev:
	Job title		
	Subject: Example- Un-Restrained Beam with end moments		
	Client: 4th YEAR- Faculty of civil Engineering	Made by: SCI	Date:
		Checked by: Dr. G. Hallak	Date:

The beam shown in Figure 3.1 has moment resisting connections at its ends and carries concentrated loads. The intermediate concentrated loads are applied through the bottom flange. These concentrated loads **do not** provide restraint against lateral-torsional buckling. Design the beam in S275 steel.

References are to BS EN 1993-1-1: 2005, including its National Annex, unless otherwise stated

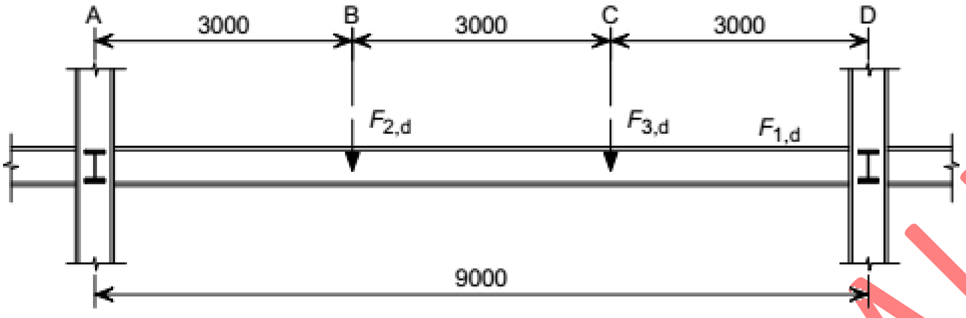


Figure 3.1

1.2 Actions (loading)

1.2.1 Permanent actions

Uniformly distributed load (including self weight) g = 3 kN/m

Concentrated load 1 $G1$ = 40 kN

Concentrated load 2 $G2$ = 20 kN

1.2.2 Variable actions

Concentrated load 1 $Q1$ = 60 kN

Concentrated load 2 $Q2$ = 30 kN

The variable actions are not due to storage and are not independent of each other.

1.2.3 Partial factors for actions

BS EN 1990 A1.3.1(4)

Table NA.A1.2(B)

Partial factor for permanent actions γ_G = 1.35

Partial factor for variable actions γ_Q = 1.50

Reduction factor ξ = 0.925

1.2.4 Design values of combined actions for Ultimate Limit State

Use Expression (6.10) or the less favourable combination from Expression (6.10a) and (6.10b). The UK National Annex to BS EN 1990 allows the designer to choose which of those options to use.

$$\gamma_{Gj,\sup}G_{j,\sup} + \gamma_{Gj,\inf}G_{j,\inf} + \gamma_{Q,1}\psi_{0,1}Q_1 + \gamma_{Q,i}\psi_{0,i}Q_i \qquad (6.10a)$$

$$\xi\gamma_{Gj,\sup}G_{j,\sup} + \gamma_{Gj,\inf}G_{j,\inf} + \gamma_{Q,1}Q_1 + \gamma_{Q,i}\psi_{0,i}Q_i \qquad (6.10b)$$

BS EN 1990 Table NA.A1.2(B)

Use 6.10b:

UDL (including self weight)

$F_{1,d} = \xi\, \gamma_G\, g = (0.925 \times 1.35 \times 3\ ) = 3.7$ kN/m

Concentrated load 1

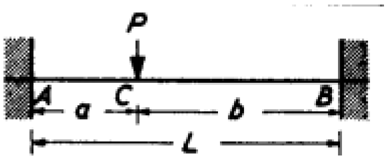
$F_{2,d} = \xi\, \gamma_G\, G_1 + \gamma_Q\, Q_1 = (0.925 \times 1.35 \times 40\ ) + (1.5 \times 60\ ) = 140.0$ kN

Concentrated load 2

$F_{3,d} = \xi \gamma_G\, G_2 + \gamma_Q\, Q_2 = (0.925 \times 1.35 \times 20\ ) + (1.5 \times 30\ ) = 70.0$ kN

1.3 Design bending moments and shear forces

Uniform $M_A = M_B = -WL^2/12$, $M_C = WL^2/24$



Concentrated $M_A = -Pab^2/L^2$, $M_B = -Pa^2b/L^2$,

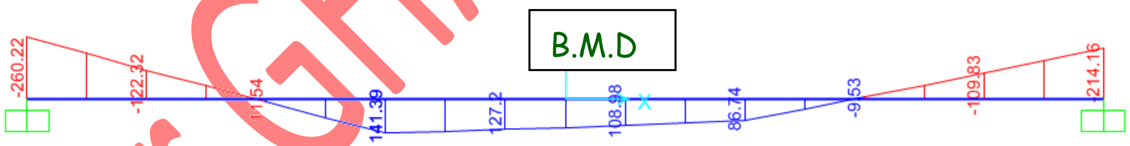
$M_C = 2Pa^2b^2/L^3$

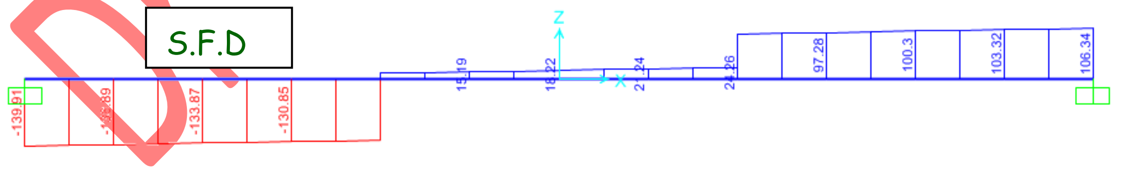
Above formulas can be used to determine the bending moments and shear forces through the beam as follows:

$M_A = -[3.7x9^2/12]-[140x3x6^2/9^2]-[70x6x3^2/9^2]=-258.3$kN.m

$M_B = -[3.7x9^2/12]-[140x3^2x6/9^2]-[70x6^2x3/9^2]=-211.6$kN.m

Or any structural analysis program can be used to obtain the internal forces diagrams as shown below





However, The design bending moments and shear forces are shown in Figure 3.2

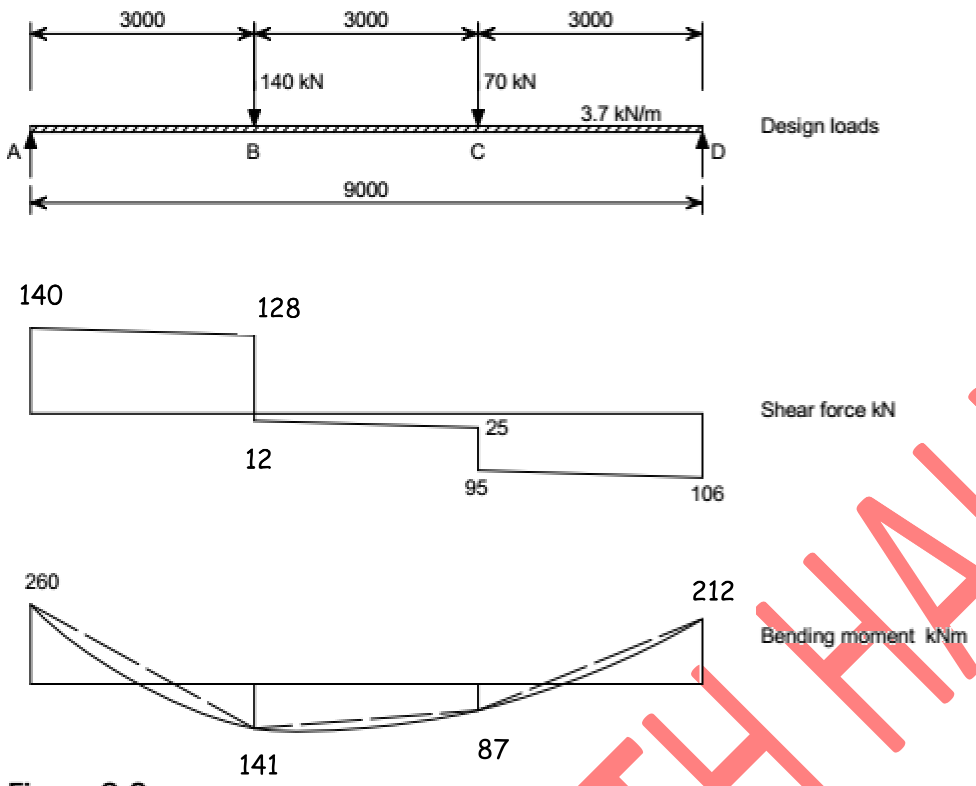


Figure 3.2

1.4 Buckling length (L_{cr})

Since the beam is unrestrained between the supports, there is only one segment to consider in this example, with a length equal to the beam length. BS EN 1993-1-1 does not give guidance for determining buckling lengths. For beams, the buckling length should be taken as being equal to the span length unless the designer considers the beam to be restrained.

L_{cr} = 9.0 m

1.5 Section properties

Trial section can be calculated as follows:

$M_{b,Rd}=\chi_{LT} W_{pl,y} f_y/\gamma_{M0}=M_{Ed}$ ⇒ $W_{pl,y}= M_{Ed}/(\chi_{LT} f_y/\gamma_{M0})$ ⇒

$W_{pl,y}$=260x106/(0.65x275)=1454cm3

Chose from the UKB section tables a section has $W_{pl,y}$ >1454cm3

Try section 457 × 191 × 67 UKB in S275 with $W_{pl,y}$ = 1470 cm3

From section property tables:

Depth h = 453.1 mm

Width	b = 189.9 mm
Web thickness	$t_w = 8.5$ mm
Flange thickness	$t_f = 12.7$ mm
Root radius	r = 10.2 mm
Depth between flange fillets	d = 407.6 mm
Second moment of area, y -y axis	$I_y = 29400$ cm4
Second moment of area, z -z axis	$I_z = 1450$ cm4
Plastic modulus, y -y axis	$W_{pl,y} = 1\ 470$ cm3
Area	A = 85.5 cm2
Buckling parameter	U=0.873
Torsional constant	$I_T = 37.1$ cm4
Warping constant	$I_W = 0.311x106$ cm6
Modulus of elasticity	E = 210 000 N/mm2

For S275 steel and $t \leq 16$ mm

Yield strength $f_y = R_{eH} = 275 \text{ N/mm}^2$

BS EN 10025-2 Table 7

1.6 Cross section classification

Table 5.2

$$\varepsilon = \sqrt{\frac{235}{f_y}} = \sqrt{\frac{235}{275}} = 0.92$$

Outstand of compression flange

$$c = \frac{b - t_w - 2r}{2} = \frac{189.9 - 8.5 - (2 \times 10.2)}{2} = 80.50 \text{ mm}$$

$$\frac{c}{t_f} = \frac{80.5}{12.7} = 6.34$$

The limiting value for Class 1 is $\frac{c}{t_f} \leq 9\varepsilon = 9 \times 0.92 = 8.28$

6.34 < 8.28

Therefore the flange is Class 1 under compression.

Web subject to bending

$c = d = 407.6$ mm

$$\frac{C}{t_w} = \frac{407.6}{8.5} = 47.95$$

The limiting value for Class 1 is $\frac{c}{t_w} \leq 72\varepsilon = 72 \times 0.92 = 66.24$

47.95 < 66.24

Therefore the web is Class 1 under bending.

Therefore the section is Class 1 under bending.

1.7 Partial factors for resistance

$\gamma_{M0} = 1.0$

$\gamma_{M1} = 1.0$ NA.2.15

1.8 Cross-sectional resistance

1.8.1 Shear buckling

The shear buckling resistance for webs should be verified according to Section 5 of BS EN 1993-1-5 if: 6.2.6(6)

$$\frac{h_w}{t_w} > \frac{72\varepsilon}{\eta}$$

η = 1.0 BS EN 1993-1-5 NA.2.4

$h_w = h - 2t_f = 453.4 - (2 \times 12.7) = 428.0$ mm

$$\frac{h_w}{t_w} = \frac{428.0}{8.5} = 50.35$$

$$\frac{72\varepsilon}{\eta} = \frac{72 \times 0.92}{1.0} = 66.2$$

50.35 < 66.2

Therefore the shear buckling resistance of the web does not need to be verified.

1.8.2 Shear resistance

Verify that: 6.2.6(1)

$$\frac{V_{Ed}}{V_{c,Rd}} \leq 1.0$$

Eq (6.17)

$$V_{c,Rd} = V_{pl,Rd} = \frac{A_V \times f_y}{\gamma_{M0} \times \sqrt{3}}$$

$A_V = A - 2t_f b + (t_w + 2r) t_f \geq \eta h_w t_w$ 6.2.6(3)

A_V=85.5×10^2-(2×189.9×12.7)+12.7×[8.5+(2×10.2)] =4093.57mm^2

$\eta\ h_w\ t_w$=1.0×428×8.5 =3638.0mm^2

Therefore,

Av = 4093.57 mm^2

$$V_{c,Rd} = V_{pl,Rd} = \frac{4093.57 \times 275}{1.0 \times \sqrt{3}} = 650.0kN$$

Maximum design shear occurs at A, therefore the design shear is

$V_{A,Ed}$ = 140 kN

$$\frac{V_{Ed}}{V_{c,Rd}} = \frac{140}{650} = 0.22 \leq 1.0$$

Therefore the shear resistance of the section is adequate.

1.8.3 Resistance to bending

Verify that:

6.2.5(1)
Eq (6.12)

$$\frac{M_{Ed}}{M_{c,Rd}} \leq 1.0$$

At the point of maximum bending moment (at A), verify whether the shear force will reduce the bending resistance of the cross section.

6.2.8(2)

$$\frac{V_{c,Rd}}{2} = \frac{650}{2} = 325\ kN$$

Shear force at maximum bending moment $V_{A,Ed}$ = 140 kN

140 kN < 325 kN

Therefore **no reduction** in bending resistance due to shear is required.(Low shear)

The design resistance for bending for Class 1 and 2 cross sections is:

6.2.5(2)

Eq (6.13)

$$M_{c,Rd} = M_{pl,Rd} = \frac{W_{pl,y} \times f_y}{\gamma_{M0}} = \frac{1470 \times 10^3 \ \times 275}{1.0} \times 10^{-6} = 404kN.m$$

6.2.5(1)
Eq (6.12)

$$\frac{M_{Ed}}{M_{c,Rd}} = \frac{260}{404} = 0.64 < 1.0$$

Therefore the bending moment resistance is adequate.

1.9 Buckling resistance of member in bending

If the lateral torsional buckling slenderness ($\bar{\lambda}_{LT}$) is less than or equal to $\bar{\lambda}_{LT,0}$ the effects of lateral torsional buckling may be neglected, and only

6.3.2.2(4)

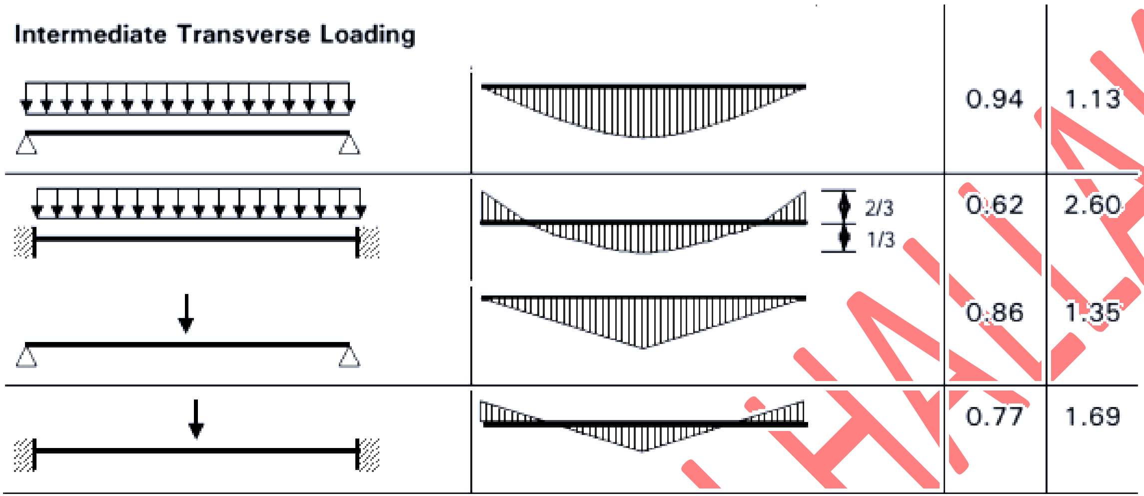
cross-sectional verifications apply

$\lambda_{LT,0}$ = 0.4 for rolled sections

NA.2.17

$$\bar{\lambda}_{LT} = \frac{1}{\sqrt{c1}} UVD\bar{\lambda}_z\sqrt{\beta_w}$$

Calculate 1/√c1



Intermediate Transverse Loading			
		0.94	1.13
		0.62	2.60
		0.86	1.35
		0.77	1.69

Take 1/√c1=0.77

U=0.873

β_w =1 class 1 & 2

$$\lambda_z = \frac{kL}{i_z} = \frac{0.7 \times 9000}{41.2} = 153$$

$$\bar{\lambda}_z = \frac{\lambda_z}{\lambda_1} = \frac{153}{93.9 \times 0.92} = 1.771$$

$$V = \frac{1}{\sqrt[4]{1 + \frac{1}{20}\left(\frac{\lambda_z}{h/t_f}\right)^2}} = \frac{1}{\sqrt[4]{1 + \frac{1}{20}\left(\frac{153}{453.4/12.7}\right)^2}} = 0.85$$

D=1 load applied at the bottom flange

$\bar{\lambda}_{LT} = 0.77 \times 0.873 \times 0.85 \times 1 \times 1.771 \times \sqrt{1}$ =1.012

The value for the elastic critical moment obtained from '*LTBeam*' is:

*M*cr = 355.7 kNm,

$$\bar{\lambda}_{LT} = \sqrt{\frac{1470 \times 10^3 \times 275}{355.7 \times 10^6}} = 1.07$$

1.012 > 0.4 ($\bar{\lambda}_{LT,0}$)

Therefore the resistance to lateral-torsional buckling must be verified

6.3.2.2(4)

$$\frac{M_{Ed}}{M_{b,Rd}} \le 1.0$$

6.3.2.1(1) Eq (6.54)

The design buckling resistance moment ($M_{b,Rd}$) of a laterally unrestrained beam is determined from:

6.3.2.1(3) Eq (6.55)

$$M_{b,Rd} = \chi_{Lt}\frac{W_y f_y}{\gamma_{M1}}$$

W_y = $W_{pl,y}$ for Class 1 and 2 cross-sections

For UKB sections, the method given in 6.3.2.3 for determining χ_{LT} for rolled sections may be used. Therefore

$$\chi_{LT} = \frac{1}{\phi_{LT}+\sqrt{\phi_{LT}^2-\beta\bar{\lambda}_{LT}^2}} \; but \begin{cases} \chi_{LT} \le 1.0 \\ \chi_{LT} \le \dfrac{1}{\bar{\lambda}_{LT}^2} \end{cases}$$

6.3.2.3(1) Eq (6.57)

$$\phi_{LT} = 0.5\left[1+\alpha_{LT}\left(\bar{\lambda}_{LT}-\bar{\lambda}_{LT,0}\right)+\beta\bar{\lambda}_{LT}^2\right]$$

From the UK National Annex, $\bar{\lambda}_{LT,0}$ = 0.4 and β = 0.75

NA.2.17

$$\frac{h}{b} = \frac{453.4}{189.9} = 2.39$$

2< 2.39 < 3.1, therefore use buckling curve 'c'

NA.2.17

For buckling curve 'c', α_{LT} = 0.49

NA.2.16 & Table 6.5

$$\phi_{LT} = 0.5\left[1+0.49(1.012-0.4)+0.75\times 1.012^2\right] = 1.034$$

$$\chi_{LT} = \frac{1}{1.034+\sqrt{1.034^2-0.75\times 1.012^2}} = 0.63$$

$$\frac{1}{\bar{\lambda}_{LT}^2} = \frac{1}{1.012^2} = 0.97$$

0.63 < 0.97 < 1.0

Therefore,

χ_{LT} =0.63

To account for the shape of the bending moment distribution, χ_{LT} may be modified by the use of a factor 'f'.

$$\chi_{LT,mod} = \frac{\chi_{LT}}{f} \; but \; \chi_{LT,mod} \le 1.0$$

6.3.2.3(2)

$$f = 1-0.5(1-K_c)\left[1-2.0\left(\bar{\lambda}_{LT}-0.8\right)^2\right] but \; f \le 1.0$$

$$f = 1 - 0.5(1 - 0.77)\left[1 - 2.0(1.012 - 0.8)^2\right] = 0.895$$

$$\chi_{LT,mod} = \frac{\chi_{LT}}{f} = \frac{0.63}{0.895} = 0.7$$

$$M_{b,Rd} = \chi_{Lt,mod} \frac{W_y f_y}{\gamma_{M1}} = 0.7 \times 1470 \times 10^3 \times 275 \times 10^{-6} / 1.0 = 283 kN.m$$

$$\frac{M_{A,Ed}}{M_{b,Rd}} = \frac{260}{283} = 0.92 \leq 1.0$$

Therefore the design buckling resistance moment of the member is adequate.

2.8 Vertical deflection at serviceability limit state

The vertical deflections should be verified.